Nonlinear Mode Coupling of a Vertically Paired Structure in Complex Plasma

REU Summer 2018

Nicholas Ernst, Zhiyue Ding & Truell W. Hyde

Overview

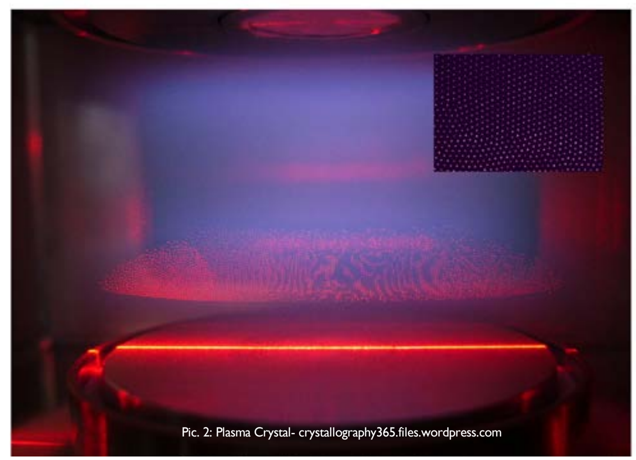
- What is Complex Plasma? (3-5)
- Our Experiment (6)
- Methods (7)
- Analysis (8-10)
- Theory (11-13)
- Conclusions and Future Work (14)
- Acknowledgements (15)

What is Complex Plasma?

www.baylor.edu/CASPER

- Dusty Plasma
- Micrometer Sized Particles Immersed in a Partially Ionized Plasma Medium
- Space, Industry, Fusion Devices, Laboratory
- Dust Exhibits Negative Charge
  - High Electron Mobility
  - Weak and Strong Coupled System
- Interesting Dynamical Behaviors
  - Crystal Formation, Wave Propagation, Fluid Flow, Phase Transitions, etc.


Pic.1: Comet- theconversation.com


Pic. 2: Plasma Crystal- crystallography365.files.wordpress.com

Complex Plasma in the Laboratory

www.baylor.edu/CASPER

Pic. 3: York University GEC Ref. Cell- www.york.ac.uk

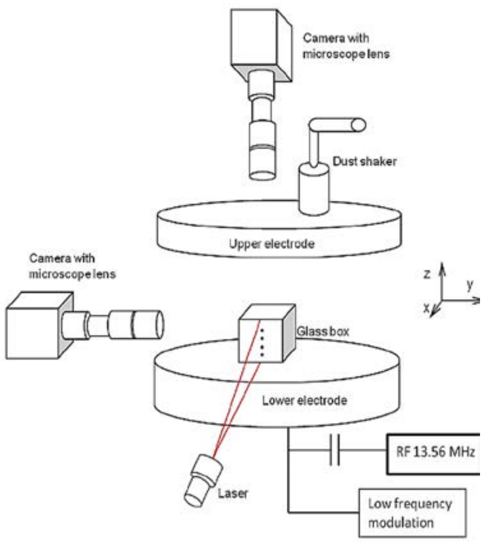


Fig. 1: GEC Reference Cell Diagram- www.baylor.edu

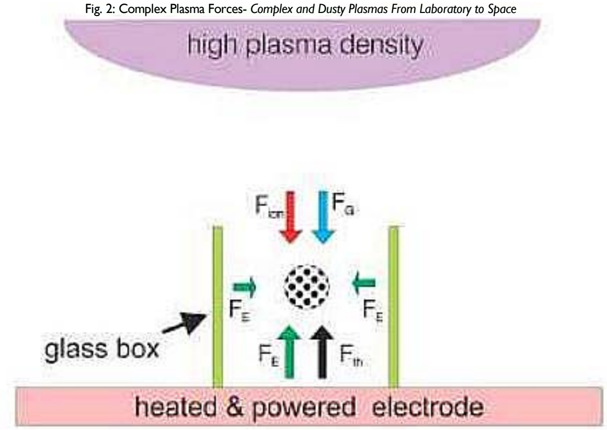
- GEC-Reference Cell
  - Capacitively Coupled RF Discharge- Argon and Neon
  - 13.56 MHz- Low Temperature Plasma
  - Negative Bottom Electrode, Grounded Top
  - Bulk Plasma and Ion Flow
  - Plasma Sheath and Parabolic Potential Well
- Time Scales Ideal for Video Microscopy
  - Laser Illumination
  - CCD Camera
- Analogue Systems

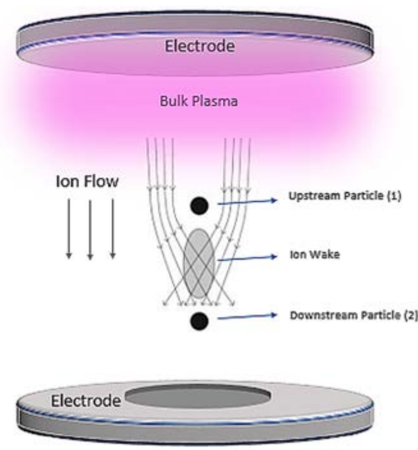
Forces and Ion Wake

www.baylor.edu/CASPER

- Gravity and Electrostatic
  - Charge-to-Mass Ratio
- Thermophoretic
- Neutral Drag
- Ion Flow
  - Ion Drag
  - Ion Wake
  - Non-Reciprocal Force
- Dynamic Response in Plasma
  - Charge, Mass, Flow Rates, Temperature, Etc.

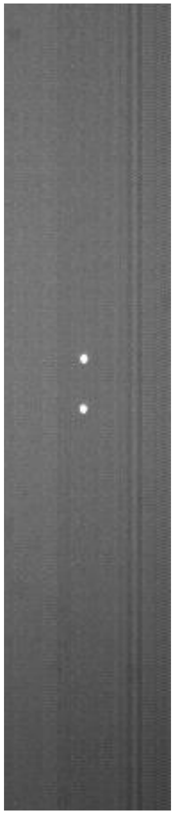
Fig. 2: Complex Plasma Forces- *Complex and Dusty Plasmas From Laboratory to Space*





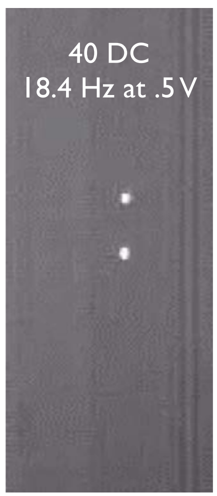
Our Work and Experiment

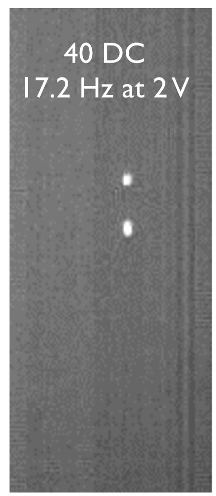
- Particle Interactions
  - Nonlinear regime
- Internal Resonance
- Two-Particle System
- Driven in the Vertical Direction
  - Modulated Sinusoidal Force
- Mode Coupling Instability
  - Vertical and Radial
- GEC Reference Cell

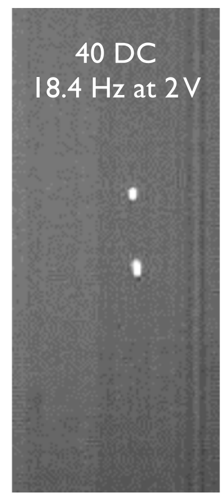


Methods

- Glass Box
- Varied: Driving Amplitude, Driving Frequency, DC Bias Voltage
- Prominent Near Breathing Frequency and High Driving Voltage

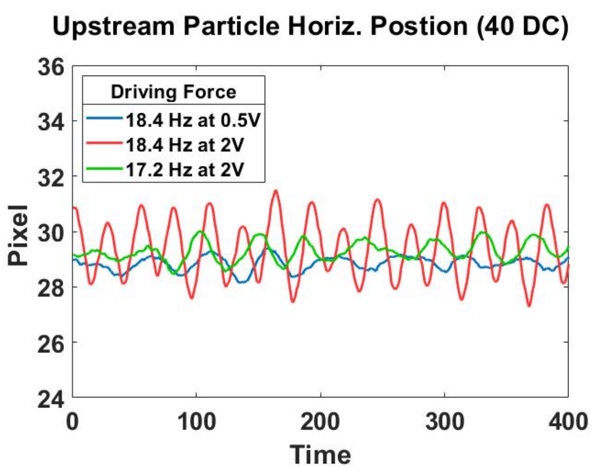


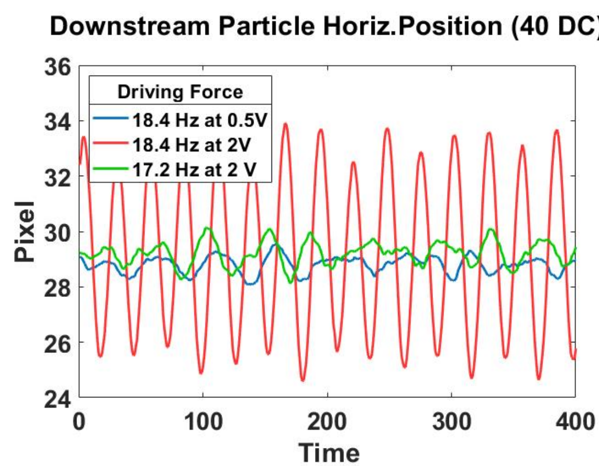




Analysis

Particle Trajectories

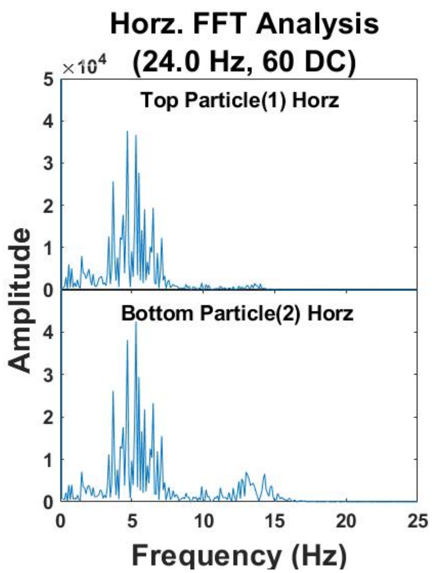


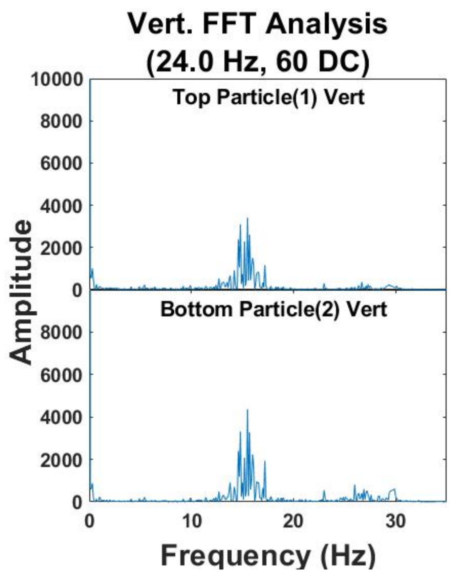


www.baylor.edu/CASPER

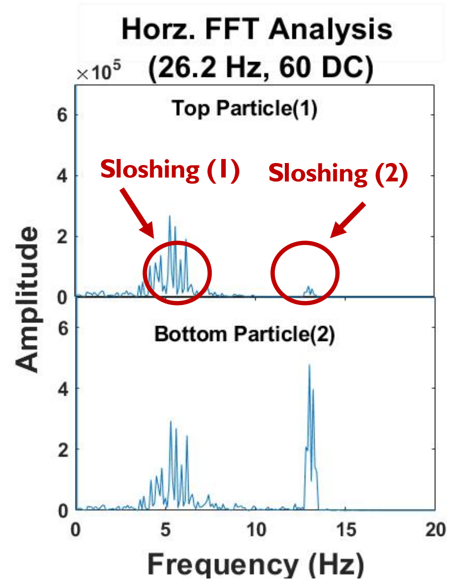
Analysis

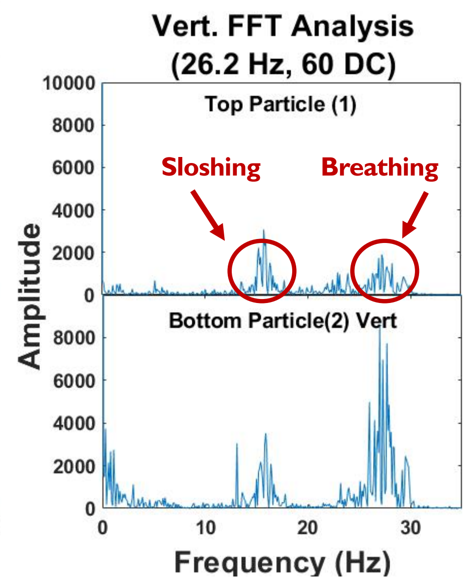
0.5 Volt Drive Amplitude





2 Volt Drive Amplitude

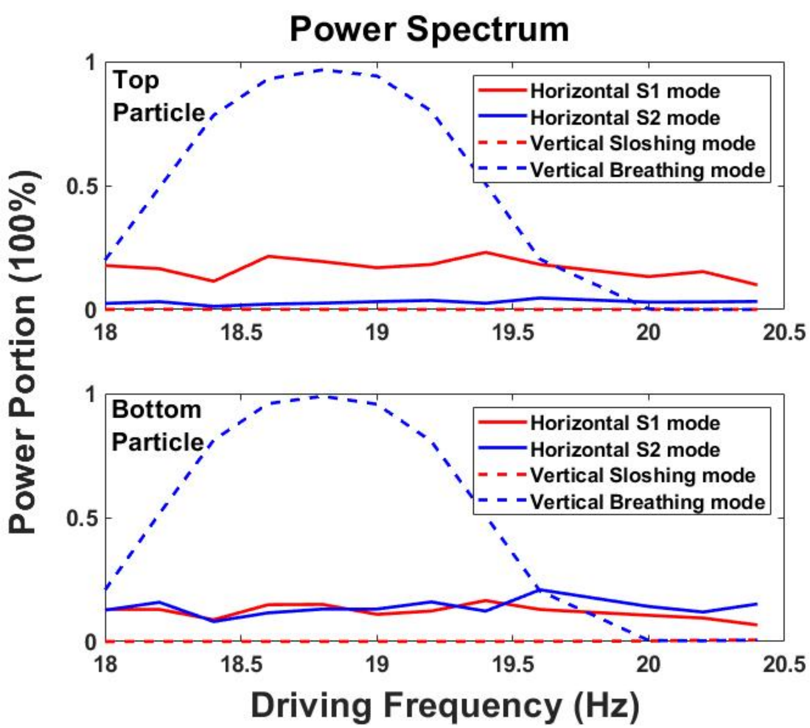




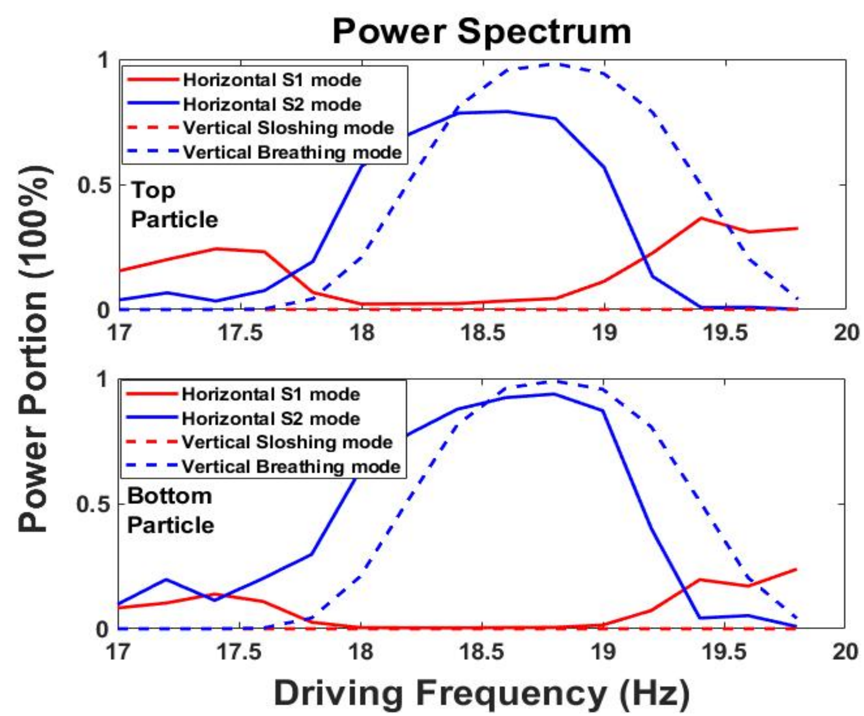
$$\textit{Sloshing (2) Frequecny} \approx \frac{1}{2}\,\textit{Breathing Frequency}$$

Analysis

40 DC at 0.5V



40 DC at 2.0V



CASPER www.baylor.edu/CASPER

Theory

- Driven, Damped Coupled Oscillator in the Linear Regime
  - Fails to Explain this Phenomena
- Other Work
  - Nonlinearities in Confinement (Anharmonic Sheath) Does Not Explain
- Impose a Force Dependent Upon Horz. And Vert. Displacement
  - Pair is Strongly Coupled System Affected by the Ion Wake
  - Further Expand into the Nonlinear Regime

Theory

www.baylor.edu/CASPER

Driven, Damped Oscillator With Non-Reciprocal Interaction Equations of Motion

$$\begin{cases} \ddot{x}_1 + \omega_1^2 x_1 + \mu_1 \dot{x}_1 + k_{12,x}(d) = 0 \\ \ddot{x}_2 + \omega_2^2 x_2 + \mu_2 \dot{x}_2 + k_{21,x}(d) = 0 \end{cases} \qquad \begin{cases} \ddot{y}_1 + \omega_3^2 y_1 + \mu_3 \dot{y}_1 + k_{12,y}(d) = F_1 e^{i\Omega t} \\ \ddot{y}_2 + \omega_4^2 y_2 + \mu_4 \dot{y}_2 + k_{21,y}(d) = F_2 e^{i\Omega t} \end{cases}$$

d- Displacement $\{(x_2 - x_1)^2 + (y_2 - y_1 + R)^2\}^{1/2}$ $F_{1,2}$- Driving Amplitude Ω- Driving Frequency

R- Equilibrium Displacement Between Particles $k_x(d) = k(d)\frac{(x_2 - x_1)}{d}$ $k_y(d) = k(d)\frac{(y_2 - y_1 + R)}{d}$

y_1- Vertical Position of Upstream Particle y_2- Vertical Position of Downstream Particle

$\omega_{1,2}$- Oscillation Frequencies k- Non-Reciprocal Force Between Particles $\mu_{1,2}$- Ion Drag Force

Interaction Force Expanded into Nonlinear Regime

$$k_x(d) \approx k_x(0,0) + (x_2 - x_1)\frac{\partial k_x}{\partial (x_2 - x_1)}\Big|_{0,0} + (y_2 - y_1)\frac{\partial k_x}{\partial (y_2 - y_1)}\Big|_{0,0} + \frac{1}{2}(x_2 - x_1)^2 \frac{\partial^2 k_x}{\partial (x_2 - x_1)^2}\Big|_{0,0} + \frac{1}{2}(y_2 - y_1)^2 \frac{\partial^2 k_x}{\partial (y_2 - y_1)^2}\Big|_{0,0} + (x_2 - x_1)(y_2 - y_1)\frac{\partial^2 k_x}{\partial (x_2 - x_1)(y_2 - y_1)}\Big|_{0,0} + \cdots$$

* $k_y(d)$ Takes Similar Form

Theory

www.baylor.edu/CASPER

- To Solve: Must Decouple Equations in the Linear Regime

New Cord. System	High Sloshing	Low Sloshing	Breathing	Sloshing
	$x_+ = x_1 - (\alpha_-)x_2$	$x_- = x_1 - (\alpha_+)x_2$	$y_+ = y_1 - (\beta_-)y_2$	$y_- = y_1 - (\beta_+)y_2$

- Our Equations Now Take The Form

$$\begin{cases} \ddot{x}_+ + \omega_+^2 x_+ + g_1\, x_+\, y_+ + g_2\, x_+\, y_- + \mu \dot{x}_+ = 0 \\ \ddot{y}_+ + \omega_{y+}^2 y_+ + a_1 x_+{}^2 + a_2 y_+{}^2 + a_3\, y_+\, y_- + a_4 y_-{}^2 + \mu \dot{y}_+ = F_1 e^{i\Omega t} \\ \ddot{y}_- + \omega_{y-}^2 y_- + h_1 x_+{}^2 + h_2 y_+{}^2 + h_3\, y_+\, y_- + h_4 y_-{}^2 + \mu \dot{y}_- = F_2 e^{i\Omega t} \end{cases}$$

- Can Now Solve Using Multiple Scale Perturbation
    - Solve to Second Order with a Fast and Slow Timescale

Conclusions and Future Work

- Experimental Analysis Points to Mode Coupling
- Our Equations of Motion Can Explain It
- Fully Solve Equations of Motion
- Explore the Peculiarities
- Run Simulations
- Perform Experiments Over Wider Range
- Look For Hysteresis
- Fundamental Insight for Particle Interactions
  - Wakefield
  - The World Operates in Nonlinear

Thank You!

- CASPER- Hypervelocity Impacts & Dusty Plasma Lab
- Baylor University
- Jorge Carmona Reyes
- Dr. Matthews and Dr. Hyde
- Zhiyue Ding

Questions?

This material is based upon work supported by the National Science Foundation And NASA Under Grants No. 1707215, 1740203, JPL1571701