Three-Dimensional Particle Tracking of a Granular System Undergoing Azimuthal Shear

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Magnetic Resonance Imaging (MRI) allows the images of the three-dimensional flow of a granular system to be optically observed in an otherwise opaque medium. Unlike other investigations, this research involves itself in the features of granular flow in a practical three-dimensional system undergoing an azimuthal shear orthogonal to gravity. Mustard seeds, nearly spherical particles, produce a clear spin echo signal for the MRI to receive, and are imaged as they lay within a bed of non-magnetic spheres. The system is examined for a relationship between particle position and behavior during shear. A comparison is also made between the mustard seed motion in different sized medium.

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Introduction

A basic understanding of granular systems is crucial to appreciating this research. A thorough background of granular systems is discussed here[1]. For brevity, we refer to this reference to summarize the complexities of the field. Granular systems are large groupings of distinct macroscopic particles. These particles are large enough to ignore thermal energy because it is overwhelmed by gravitational energy[1, 2]. A granular system may behave as a solid, a liquid, or a gas depending on the circumstances, and the thresholds between states are dependent upon parameters other than temperature. When these particles do not stick together they repel one another, thus they form no definite shape as a whole, but settle into a stationary mound[2]. Another key characteristic to understanding granular material is that interactions between particles are inconsequential due to static friction and inelastic collisions. In addition, granular medium tend to segregate in conditions where most matter mixes[3]. A perfect example of this is the Brazil Nut Effect (BNE)[4].

The BNE highlights the motion of an intruder particle larger than the local medium. This larger particle will surface through the medium during vertical vibration in 3D, or gradually relocate to the edge of the system in a 2D system undergoing swirling. The corresponding Reverse Brazil Nut Effect calls attention to a massive intruding particle's ability to sink to the bottom in the same vertical vibration[5], or move to the center of the identical swirling cell[4].

Much study regarding the flow of a granular system under shear has taken place in an effort to find information on the nonlinear dynamics of the system. The difficulty behind understanding granular materials is due to them not having general equations of state, besides Newton’s laws, much like solids, liquids, and gases do. Interesting properties are observed when only considering the system as one state of matter or as a mixture of state. For example, resting granular materials show no height-dependent pressure. The reason for this is that particles distribute their weights randomly and unbalanced.

Granular constituents rest like solids but flow like liquids in models referred to as granular hydrodynamics (GH). The Navier-Stokes equations for fluids use an averaging process that is not consistent to what occurs in GH, therefore the equations do not hold in these cases[2]. Also, a major difference between granular materials and ordinary fluids is the presence of shear bands[6]. Dense slow flows occur simultaneously with rapid gaslike flows, which make relationships very mathematically involved.

As a group of free individual particles, the granular system is at liberty to behave as a gas. Unfortunately, the interactions between particles are essentially inelastic therefore energy is lost during each collision. As a result, standard theories for ideal gases are not valid. An interesting type of particle interaction happening only in granular materials is “inelastic collapse,” where clustering occurs due to an infinite number of collisions in a finite time.

In an attempt to understand more, research of granular systems has involved itself in the third dimension. Valuable information has helped explain the properties of the material through this extra dimension, especially regarding shear. Allowing flow in the third dimension affects the flow traits of the medium.

(i) A larger amount of frictional forces are present deeper in the medium[7]. More particle interactions occur in 3D medium and result in a larger quantity of normal forces. The total of all the frictional forces associated with the normal forces makes particles more resistant to shear.

(ii) Particles in different positions of the material do not travel the same distances as other particles because the normalized tangential velocity of each particle decreases strongly with distance from a moving wall[8]. Not only does the tangential velocity change, but the angular velocity can vary throughout the medium depending on the shear style[6]. Cheng *et al.* rotate a disk underneath the medium. They state that the angular velocity of particles at the center of the cell stays constant for small stacks of material, but starts decreasing after the critical height they found to be approximately $.6R_s$. Also, particles in the center of the system require a larger box aspect ratio (L/H) to become involved within shear flow than a particle towards the boundary[9].

(iii) Motion is not always constant due to a phenomenon called Stick-Slip Motion. There are three parts to this observation[10]. The first part, elastic loading, occurs when stress chains form from the drive displacement. Then there is the plastic creep where the granular assembly expands slightly from some sort of stress chain rotation. Finally, brittle failure occurs when the granular material can no longer support the force generated by the shear.

Another key component to this research lies in the shearing technique. Previous procedures have focused on 2D shear in one direction[4, 7], 3D shear with tilting walls[11]

[Pouliquen], and 3D shear with a Couette or modified Couette system[2, 6]. A standard Couette system is a two-cylinder apparatus. The smaller cylinder is placed in the middle of the larger cylinder, and the medium rests in between the walls of the two cylinders. Then one or more of the cylinders is used to shear the walls in a circular motion. Unlike most shear methods, our granular media experiences azimuthal shear orthogonal to gravity from the vertical walls. An inverted Teflon cup rests above the material and spins about its center directly shearing the sidewalls of medium. Our research investigates the behavior of this immediate layer and reaction of the whole system to this shear. The hope for the review of the sheared system is to develop a relationship between the position of the tracer particle in the system and the behavior of the particle.

Method

The MagneVu 1000 MRI machine has a viewing area of 50x75x10 mm and has the option to create “T1” or “STIR” images. Images are produced using the “T1” option because these representations tend to have better image resolution. The mustard seeds have an average diameter of 2.01 mm and an average mass of .005 g. A majority of the data taken is with a medium consisting of glass spheres with a diameter approximately 1.85 mm to establish a basis for shear flow. The second part investigates the results of changes in media size by replacing the glass spheres with Delrin spheres 2.38 mm in diameter and a mass of .00929 g. This alteration changes the tracer to media size ratios and the behavior is observed.

A ramification for using MRI is that nothing magnetic can be used in the collection of data. The cell machined for this project is made of a hollowed out Delrin cup 1.875 in deep and with an inner radius of 1 in. A thin Delrin rod is then inserted through the top of a caste acrylic lid via a glass bearing and attached to an inverted cup made of Teflon. The inverted cup is .6 in deep with an inner diameter of 1.950 in and contains the granular sample. The rod is then rotated to spin the inverted Teflon cup, which shears the walls of the granular material. The MRI creates an image in between the 60-degree counter-clockwise rotations of the inverted cup. Previous studies have shown that the stopping and starting of the shear does not play a role in the results[6] [Cheng,].

The MRI analyzes a 3D volume and creates a 3D array of voxel size (85, 128, 25) that can be broken down into cross-sections in Interactive Data Language (IDL) that "stack together" to fabricate the entire image. Therefore, the 3D images can be analyzed as a series of connected 2D representations. Although the images in the stack are 2D, each position in the image has a pixel value, and plotting the image with pixel intensity produces a third-dimension that aids particle identification analysis, and contours can then be created from a surface shading of the image. The purpose of these contours is to isolate particles from the rest of the image, so an intensity level must be specified for each contour to avoid the image's noise. To more easily find an acceptable intensity level the pixel values are manipulated by subtracting the mean value of the noise. This places most of the non-particle positions near a pixel intensity of zero. Since the image acts like an array, it is multiplied by a scalar to stretch the peaks even further from the noise.

Once the contours distinguish only particles, each position along the contour path is sent to the moment function and evaluated by the first and second moments.

$$\bar{x} = \frac{1}{N}\sum_{j=0}^{N-1} x_j \qquad (1)$$

$$\frac{1}{N-1}\sum_{j=0}^{N-1}\left(x_j - \bar{x}\right)^2 \qquad (2)$$

The first moment calculates a mean value of all positions in each direction while the second moment finds the variance of each position from the average. The variance is used as a check to make sure that the contour found is large enough to represent a particle.

This process begins in the XY plane and creates an XY coordinate pair that represents the center of mass of each particle. The program then searches through the YZ plane at the X value determined earlier and outputs a YZ center using the same method. Similarly, the XZ plane outputs an XZ center. The center values are then averaged and stored. This action occurs each time a high-intensity pixel grouping is recognized, and because the same mustard seed appears throughout multiple z-slices, multiple values for the center of mass are created. These values are then grouped together for one final average.

As far as tracking particle motion is concerned, consecutive stacks of images are compared. The program prompts the user to match the first set of particles to their corresponding positions in the second set. After that, a displacement is created to utilize Nicolas Oullette's, a professor at Yale University, method mentioned in *Experimental and Computational Techniques in Soft Condensed Matter Physics*[12]. He optimizes the correspondence of particles from one image to the next by finding a minimum total cost

$\sum_{n,i,j}\phi_{ij}^{n}$ of matching particles together for cost function ϕ_{ij}^{n}, over *n* images ranging from *i* to *j*. Using Oullette's two-frame predictive algorithm along with a cost function similar to above, the cost function is now defined as

$$\phi_{ij}^{n} = \min_{k}\left|\tilde{x}_{i}^{n+2} - x_{k}^{n+2}\right| \tag{3}$$

where $\tilde{x}_{i}^{n+2}$ is the estimated particle position in frame n+2, and x_{k}^{n+2} is the actual position of a particle in that same frame. For multiple particles, the optimized combination of matching particles occurs with the smallest total cost function.

The accuracy of the tracking algorithm is displayed in the ratio

$$\xi = \frac{\Delta r}{\Delta_0} \tag{4}$$

where Δr is the mean distance one particle has moved between images and Δ_0 is the smallest distance between particles of the same image. Tracking becomes nearly impossible as ξ becomes larger. In fact, for $\xi \approx .5$ this algorithm has a failure rate around 10%.

Results

Figure 1 represents the reproducibility of the azimuthal shearing with the glass sphere medium. The mean flow of each data set stays relatively constant showing that if dispersion occurs, it occurs consistently.

This shearing technique reveals itself to be uniform throughout the entire system when the particles and medium are approximately the same size. Figure 2 shows the particle's change in theta is unchanging. Omitting scatter, the particle rotates within the range of 0 to -.2 radians each shear. A particle's vertical height does not affect its rotational ability, therefore a particle will act similarly regardless of height. The range of rotation comes as no surprise since the walls were consistently rotated 60 degrees counter-clockwise and the particles themselves moved only a fraction of the wall's motion due to individual rotations and interactions between particles. Figure 3 exhibits a similar angle change independence. A particle's change in theta falls within the same range mentioned earlier without consideration of its proximity to the shearing wall. The motion of the particles in this scenario demonstrates uniformity because a particle has a constant angle change for each shear regardless of vertical or horizontal position.

The actual shearing characteristic within the media is indicated in Figure 4. Particles undergo more vertical motion when their distance from the wall is minimal.

The flow characteristics changed when the medium was switched to the Delrin spheres with a diameter of 2.38 mm. Figures 5 and 6 present evidence that a granular system of different sized particles does not flow the same when undergoing shear. As shown in Figure 5, the horizontal distance between a particle and the center of the cell fluctuates depending on the angular position of the particle. This suggests a lack of uniform motion when this system undergoes shear. This suspicion is only strengthened in Figure 6, because the particle's change in vertical position seems to rely on the horizontal distance between the particle and the center.

Conclusion

This research reports that there exists a relationship between media particle and tracer size that affects the motion of the system when encountered by an azimuthally shearing wall. By MRI, the particle motion in 3D appears to be relatively uniform when the tracer particle is approximately the same size as the glass medium. In this scenario, shear effects can be noticed with particle positions close to the shearing wall. The uniformity within the granular material is lost when the medium changes from glass spheres of diameter 1.85 mm to Delrin spheres of diameter 2.38 mm. There exists some factor that resulted in particle behavior dependent upon the angle and radial position of the particle when the mustard seeds were smaller than the Delrin spheres.

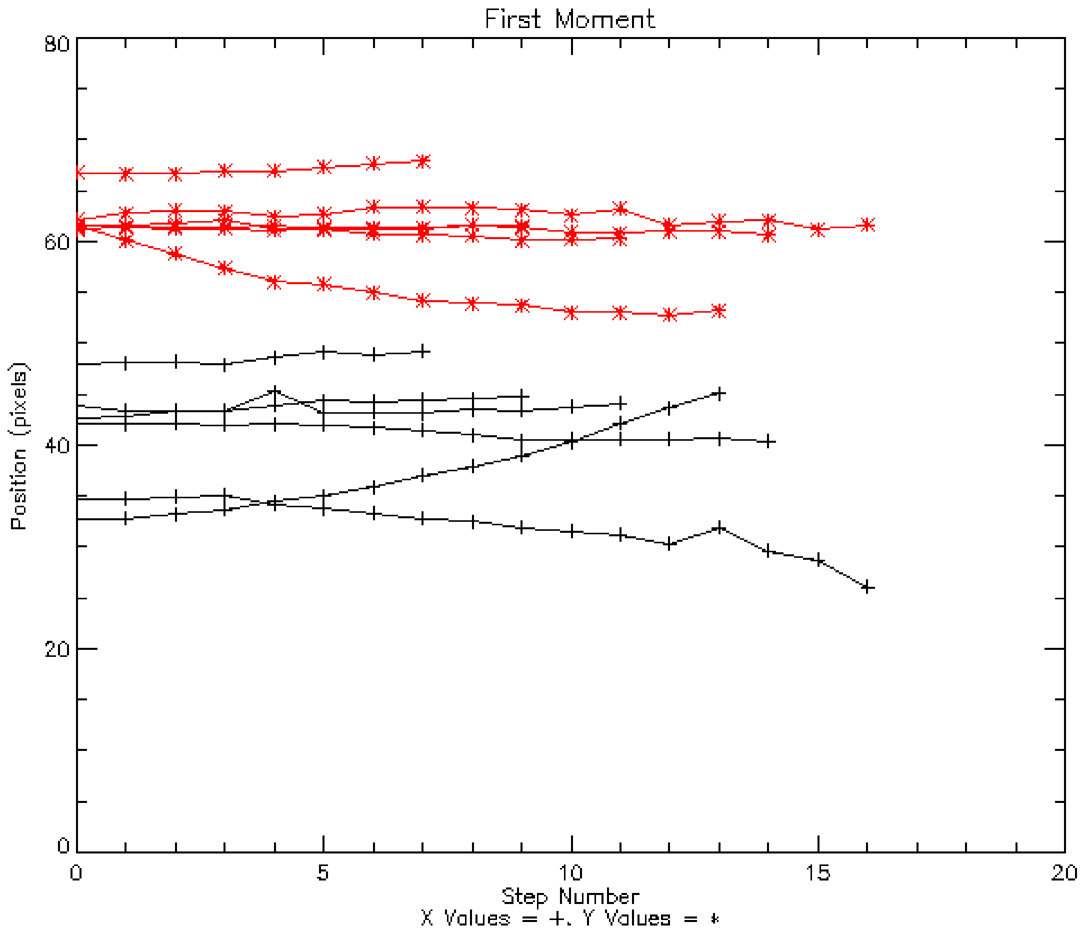

Figure 1: A plot showing how the data is reproducible. The Mean Flow of the particles in a system stays relatively constant throughout the different trials when the medium consists of glass spheres with a diameter equal to 1.85 mm. The red and black illustrate motion in the y- and x- direction respectively.

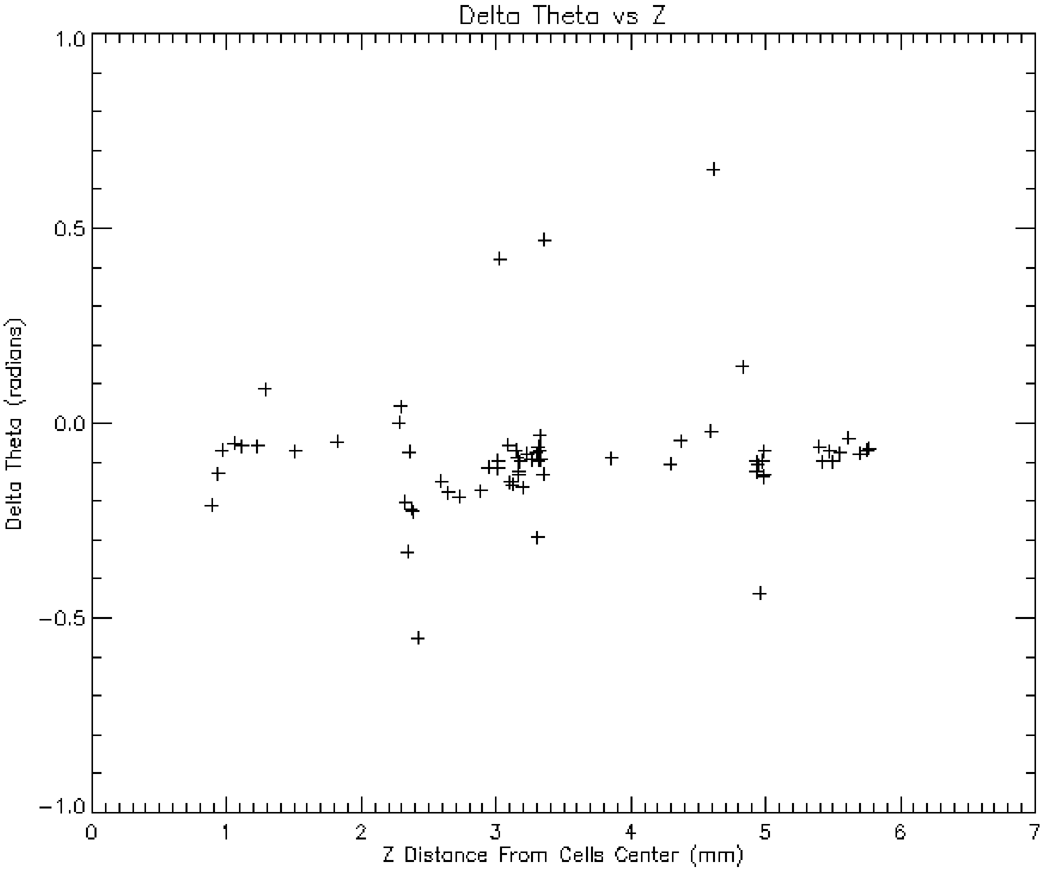

Figure 2: A comparison in cylindrical coordinates of the change of a particle's angle position with respect to a vertical distance from the center of the material in a glass sphere medium with an average diameter of 1.85 mm.

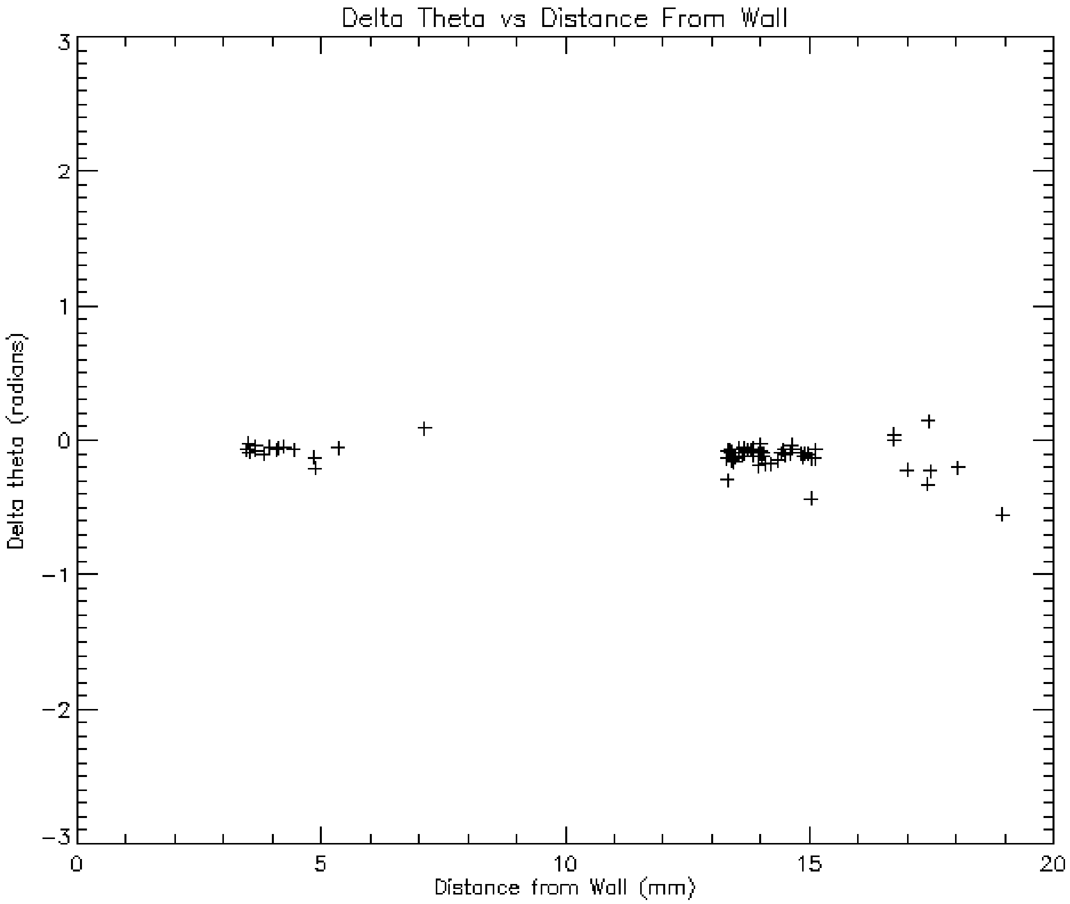

Figure 3: A comparison in cylindrical coordinates of the change of a particle's angle position with respect to a horizontal distance to the shearing wall in a glass sphere medium with an average diameter of 1.85 mm.

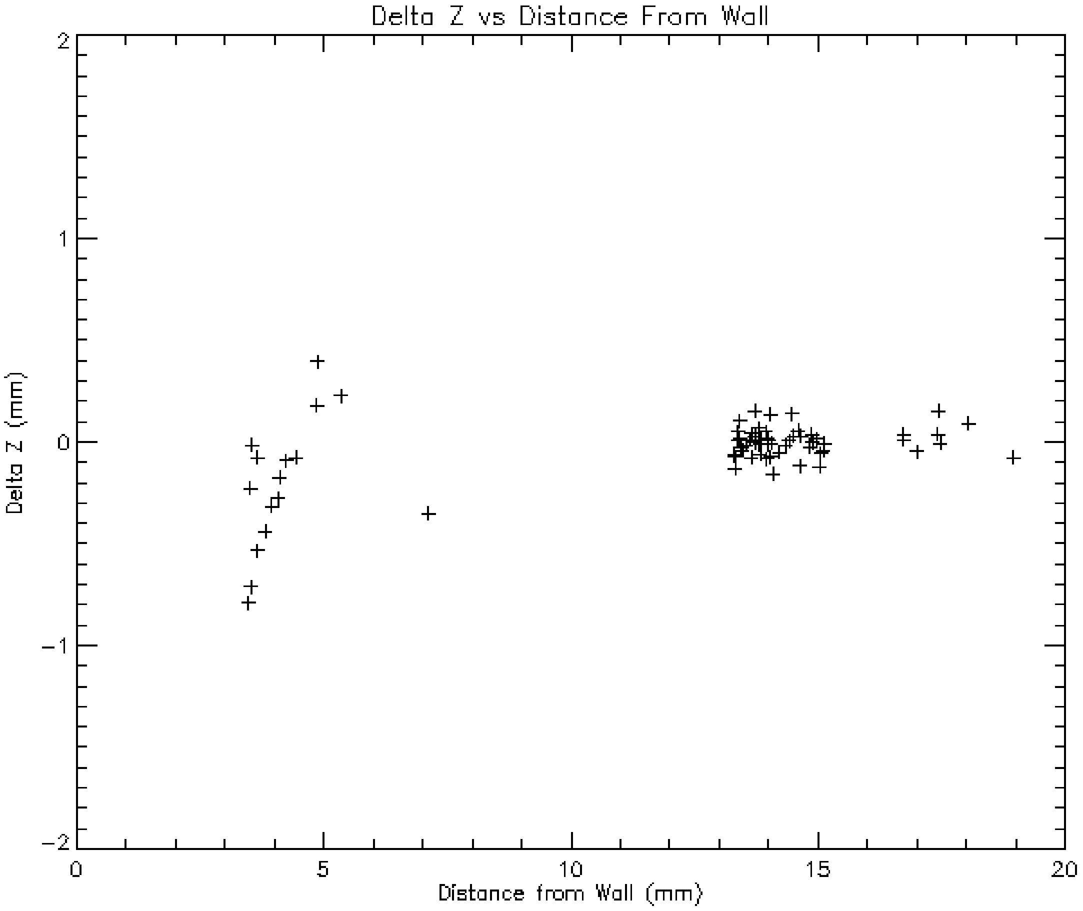

Figure 4: A plot of a particle's change in height with respect to its proximity to the wall for glass sphere medium with an average diameter of 1.85 mm.

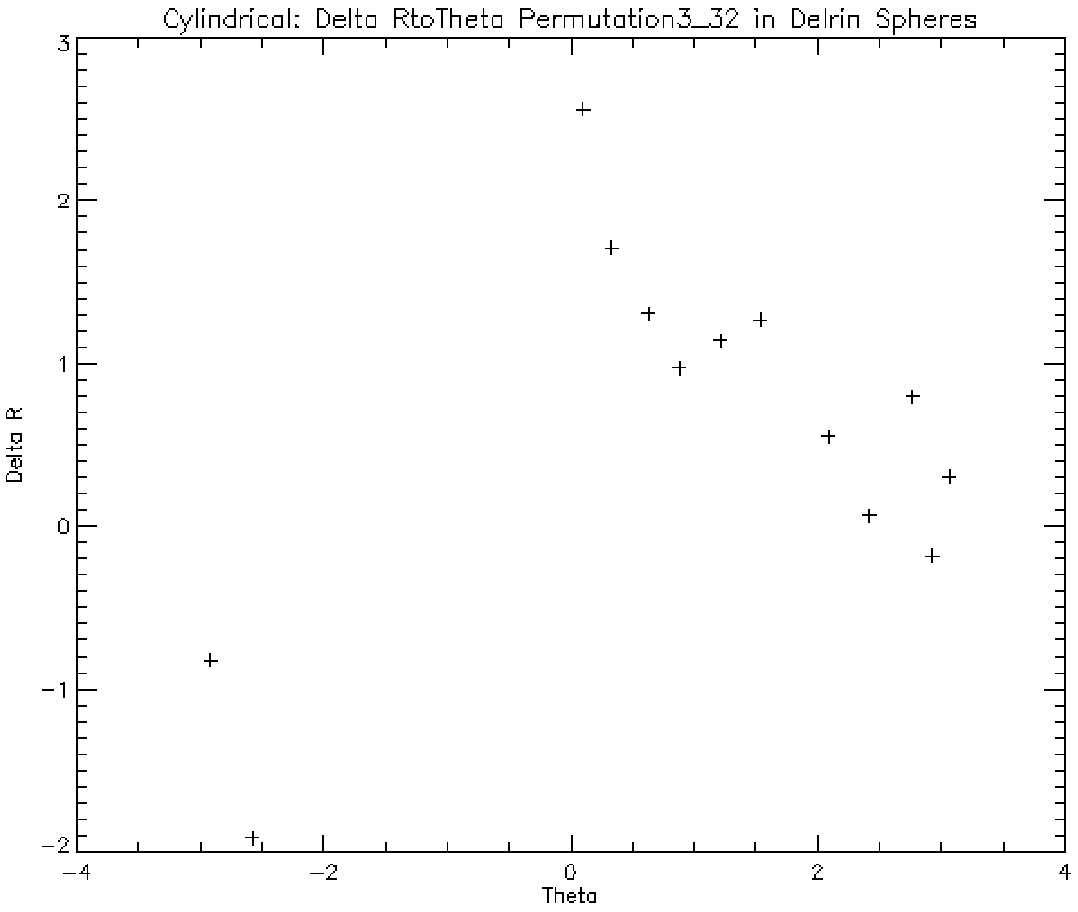

Figure 5: A plot showing the change in the distance from the particle to the center of the cell with respect to the angle position of the particle in the larger Delrin Spheres with a diameter of 2.38 mm.

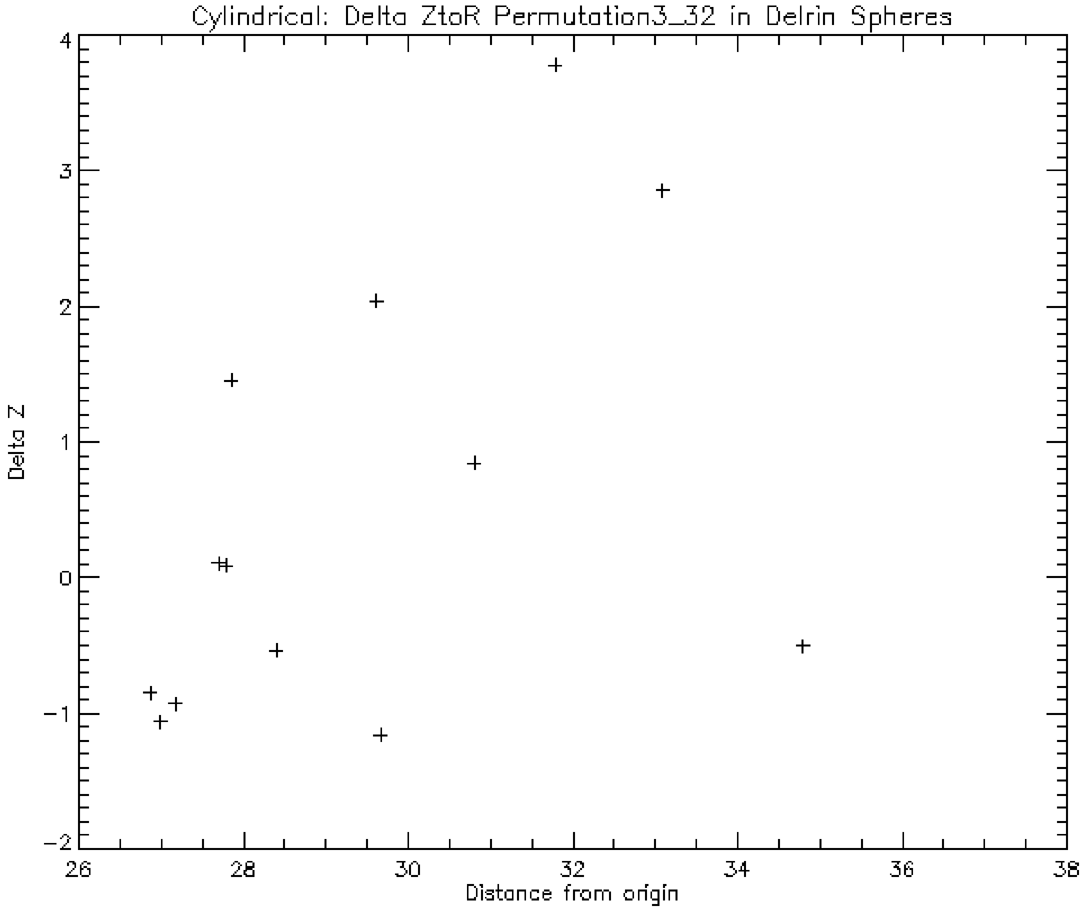

Figure 6: A plot showing the change in the vertical height of the particle with respect to its horizontal distance to the center in the larger Delrin Spheres with a diameter of 2.38 mm.

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