Simulations in Saturn's F Ring

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Abstract:

While Saturn's rings are well known, the F ring in particular is full of interesting and dynamic structures, but they are as yet largely unexplained. The F ring has been recorded having a single core with an envelope of dust, several individual strands, several intertwined strands, or clumps. This simulation attempts to explain the structures by modeling charged particles under eight different forces and examining their orbits to see if any of these structures arise. The simulation takes into account Saturn's gravity, Saturn's magnetic field, the gravitational interaction from both Prometheus and Pandora, solar gravity and the solar radiation force, and both the Coulomb and gravitational interparticle forces. As it is computationally unreasonable to model dynamics of a large number of individual particles, superparticles were used to simulate more particles, to the extent that the interparticle forces were significant.

Introduction:

Saturn's rings are well known as an exciting feature of the solar system. The F ring, located just outside the A ring, is not as well known as the others, but is far more interesting and dynamic [1]. The F ring shows a number of unusual features, including strands, clumps, braids, and spokes. There is yet no complete explanation for these different features, although two of Saturn's moons are believed to be at least partially responsible. Previous research has shown that there is a plasma environment around Saturn [2], which means that the particles in the F ring are likely charged due to collisions with the charged particles in the plasma. These charged particle interactions – both with Saturn's magnetic field and with each other – provide a likely explanation for the odd structures found in the ring, but has not been confirmed.

Forces Studied:

No single individual force can be responsible for the full array of phenomena observed within the ring. The simulation considered eight forces which are significant. The most obvious and strongest force on the particles is the gravitational force from Saturn, which keeps the particles in their orbit. The equation for gravitational force is well known:

$$F_G = \frac{GM_1M_2}{r^2}$$

where G is the gravitational constant, M_1 is the mass of Saturn, M_2 is the mass of the superparticle, and r is the distance between them. However, it is not enough to account for any of the interesting structures. For a superparticle of mass $3.91e^{-6}$ kg, the force due to Saturn's gravity is on the order of 10^{-6}N. Other gravitational forces include the force due to both moons and the interparticle gravitational force, as well as a component of the solar force. These forces were calculated for the initial position of the superparticle at a radial distance of $1.40e^{8}$ m from Saturn, and a theta position of 0 rad. Saturn's mass is taken to be $5.69e^{26}$ kg. The mass of the sun used is $1.99e^{30}$ kg and the radial distance from the sun is $1.47e^{12}$ m. Prometheus's mass is $5.80e^{-10}$ kg, and Pandora's mass is $3.42e^{-10}$ kg. Prometheus's orbit has a semi-major axis of $1.39e^{8}$ m and an eccentricity of $1.92e^{-3}$, and started the simulation at an angle of 5.92 rad. Pandora's orbit was characterized by a semi-major axis of $1.42e^{8}$ m, an eccentricity of $4.37e^{-3}$, and a starting angle of .50 rad. The force due to the moon Prometheus is on the order of 10^{-11} N; the force due to the moon Pandora, $10e^{-14}$ N, and the interparticle gravitational force is on the order of $10e^{-33}$ N. Saturn's gravity keeps the particles in their orbit, but does not provide a perturbing force that can explain the structures found in the ring, though it is the strongest of the forces acting on the superparticle.

The most obvious perturbing force on the particles is the shepherding moons. The F ring has two shepherding moons, Prometheus and Pandora, which interact with the ring at regular intervals and are gravitationally affecting the particles. Prometheus, being both larger and closer to the ring, has a stronger effect. Observations have shown that Prometheus produces "fringes" within the ring, as seen in Figure 1 [3]. These fringes are interesting, but too consistent and regular to be the sole cause of the different phenomena, as the fringes happen at regular intervals and the different structural phenomena do not seem to follow such a consistent pattern. The effect from Pandora is smaller and less obvious, but its proximity to the ring makes the force still significant.

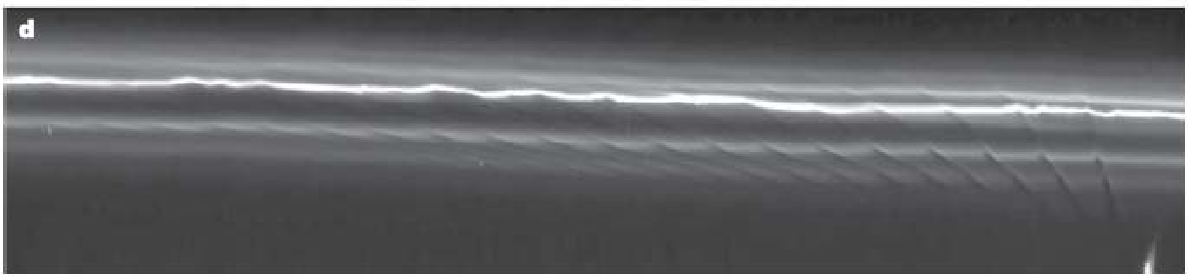

Figure 1: An image of the fringes in the F ring caused by Prometheus. Prometheus can be seen in the bottom right-hand corner of the image.

Another source of perturbing force is the sun. The sun actually supplies two forces, conveniently on the same order of magnitude. Solar gravity is simply the gravitational pull of the sun on the individual dust particles. Solar radiation pressure is the momentum transfer that results from the radiating photons from the sun impacting the dust particles. The dimensionless characteristic β is used to characterize the solar radiation pressure [4]:

$$\beta = \frac{F_{rad}}{F_{grav}}$$

$$F_{sol} = (\beta - 1) \times F_{grav}$$

(F_{grav} is calculated with the Equation 1, only with M_1 equaling the mass of the sun instead of the mass of Saturn.) The solar force on a superparticle with a typical value of $\beta = .2$ is about 10^{-10} N.

Furthermore, the particles are charged. This means that they interact with Saturn's magnetic field in a direct relationship to their charge-to-mass ratio (C/kg) [2]. The Lorentz force,

$$F = q[\vec{E} + (\vec{v} \times \vec{B})]$$

describes the force exerted on a moving charged particle by a magnetic field, where *q* is the total charge on the superparticle, *v* is the velocity, *B* is the magnetic field, and *E* is the electric field. The force from Saturn's magnetic field on a superparticle is 10^{-10} N. It has previously been shown that particles with a q/m larger than ± 0.03 C/kg will be ejected from the ring by the Saturn's magnetic field. [2]

The particles within the F ring will also interact with each other. While the particles are spaced far enough apart that actual collisions are rare [1], they still interact through Coulomb repulsion and gravitational attraction. These forces are smaller than the others with the larger of the interparticle forces being ten thousand times smaller than the smallest of the non-interparticle forces, but may provide the key to understanding the odd phonemena observed in the ring.

Since all the particles are charged in the same plasma environment, they all charge to the same potential [5]. Therefore the Coulomb forces are all repulsive. The Coulomb repulsion between two identically charged (q/m = -0.02) superparticles close to each other in the ring (radial difference about π/50) is about 10^{-19} N.

The last force taken into account is the force of gravitational attraction between the particles. Using two identical superparticles of mass $3.91e^{-6}$ kg and radial distance of π/50, the gravitational attraction is on the order of 10^{-33} N.

Method:

A numerical simulation was used to study the ring structure. The program was run in MATLAB, which allowed for easier debugging and the importation of previous pieces of code. The goal of first part of this project was to determine which kinds of particles can exist in the F ring. The simulation was run for 40 years, and six different orbital elements were graphed. By examining the graphs, it is possible to determine if a particle stayed in the F ring over the course of the simulation, or if the different forces combined to adjust its orbit outside the range of the F ring. This kind of simulation has its limitations, however, in that it is difficult to draw conclusions about the structure of the F ring when examining only one particle at a time.

However, it is computationally inefficient to attempt to track many different particles. To work around this limitation, the simulation was run with "superparticles" – clouds of many particles. These superparticles are treated as single particles by MATLAB, but have the charge and mass of many individual particles (a thousand). By tracking the superparticles, the program effectively simulates thousands of particles. Each superparticle is conceived to be a cloud of identical particles, and the program tracks the center of mass of the cloud. Superparticles also allow the program to take into consideration the interparticle forces (Coulomb repulsion and gravity) that would otherwise be too small to consider but may be large enough to affect structures in the ring.

Results:

The particles were graphed in sets of four to make them easier to compare. Each set corresponds to a given q/m ratio, with the four different β values being graphed simultaneously in different colors. In Figure 2, the β = 0.2 particle is graphed in red, the β = 0.4 particle is graphed in black, the β = 0.6 particle is graphed in green, and the β = 0.8 particle is graphed in blue.

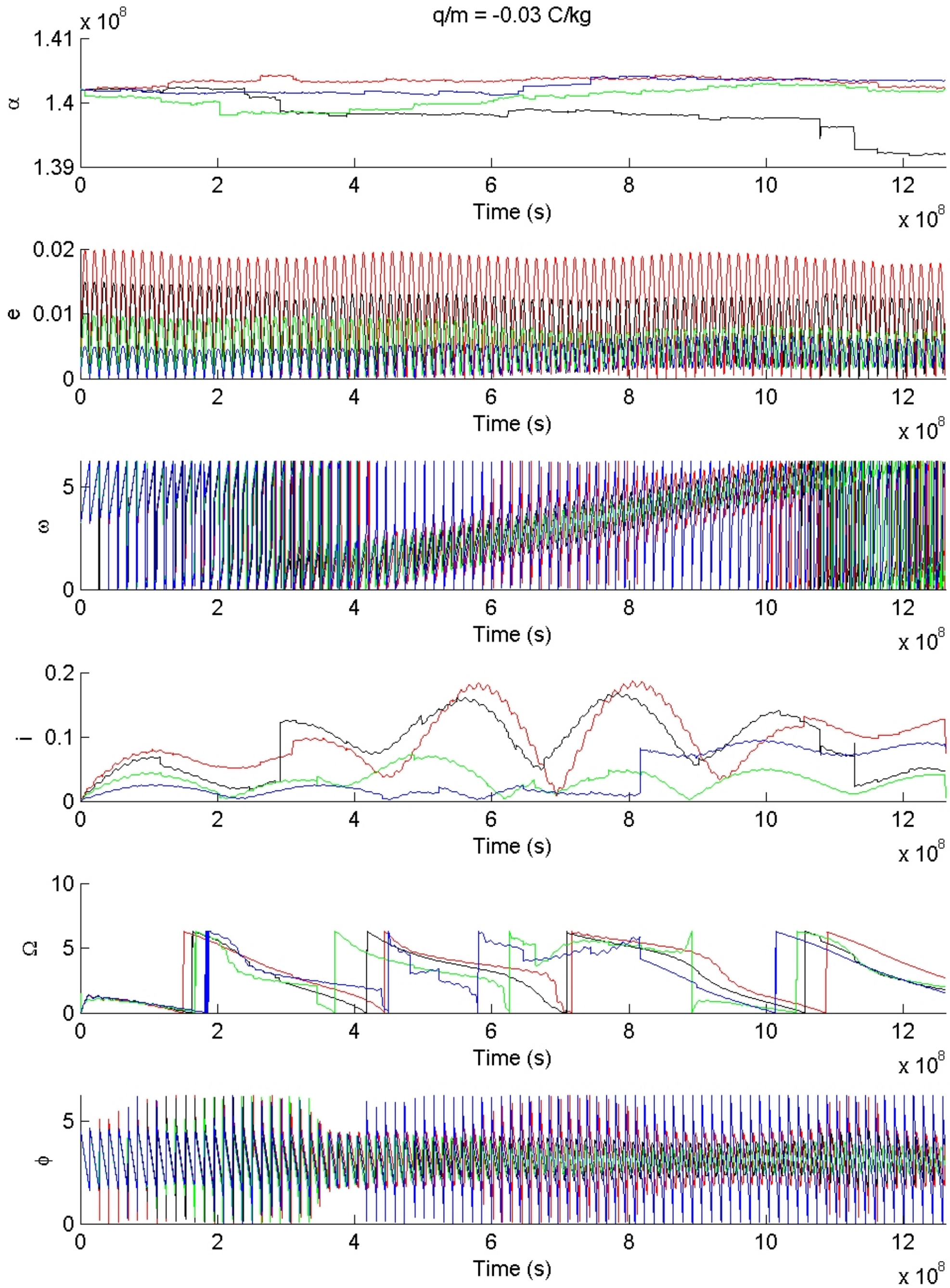

Figure 2: Graphs of the six orbital parameters for a set of particles with varying β and constant q/m = -0.03: (a) semi-major axis, (b) eccentricity, (c) argument of pericenter, (d) angle of inclination, (e) longitude of ascending node, and (f) solar angle

Discussion:

Examination of the graphs reveals that some of the particles leave the F ring. As the F ring is very thin (50 km from the core to the outside of the envelope), even a small deviation can result in a particle being ejected from the ring. Table 1 summarized which particles were and were not still in the F ring after the simulation time had finished.

Table 1	β = 0.2	β = 0.4	β = 0.6	β = 0.8
q/m = -0.03	Yes	No	Yes	No
q/m = -0.02	Yes	Yes	Yes	No
q/m = -0.01				
q/m = 0				
q/m = 0.01	No	No	No	Yes
q/m = 0.02	No	No	Yes	Yes
q/m = 0.03	No	No	No	Yes

Table 1: A summary of which particles remained in the F ring after a 40-year simulation.

Table 1 shows a tendancy for negatively charged particles in the F ring to have small β values, while positively charged particles in the F ring have large β values.

The simulation code for the superparticles is finished, but still requires some testing before data can be run with it.

Future Work:

The superparticle simulation still needs to be tested to make sure all of the smaller forces are perturbing the particles the way they should be. Then data can be run with it. The single particle code could be run for particles starting in different places in the ring, or in different relative locations to the moons.

Acknowledgements:

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References:

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