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Normal Mode Analysis of Chain Structures in Complex Plasma

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Introduction

A complex plasma consists of dust particles immersed in a weakly ionized gas, a common occurrence in astrophysical environments. In typical laboratory conditions particles obtain large charges and become strongly coupled via the screened Coulomb force. Plasma crystals have been studied extensively since their 1994 discovery [1]. Dust particles have large enough length and time scales to be observed through video microscopy, a method which does not perturb the system. Complex plasma particles are ideal models for the study of classical and condensed matter systems.

Systems with low particle numbers may be characterized by normal vibrational modes [2]. These mode eigenvectors and corresponding eigenfrequencies are determined by solving the linearized equation of motion

$$M\ddot{\vec{r}} = D_{\alpha i,\beta j}\vec{r}$$

with $\alpha,\beta = x,y,z$ and $i,j = 1,2\ldots,N$. Here the dynamic matrix is defined as

$$D_{\alpha i,\beta j} = \left(\frac{\partial F_{\alpha i}}{\partial r_{\beta j}}\right)$$

In typical laboratory conditions the path of ions are bent as they flow past the charged particle, leading to a complex downstream wake potential. The goal is to obtain a qualitative description of the potential for a single particle chain purely by comparing theoretical eigenfrequencies to experimental mode spectra. An additional goal is to explain the mode spectrum of a hexagonal chain structure.

Setup

(a (b

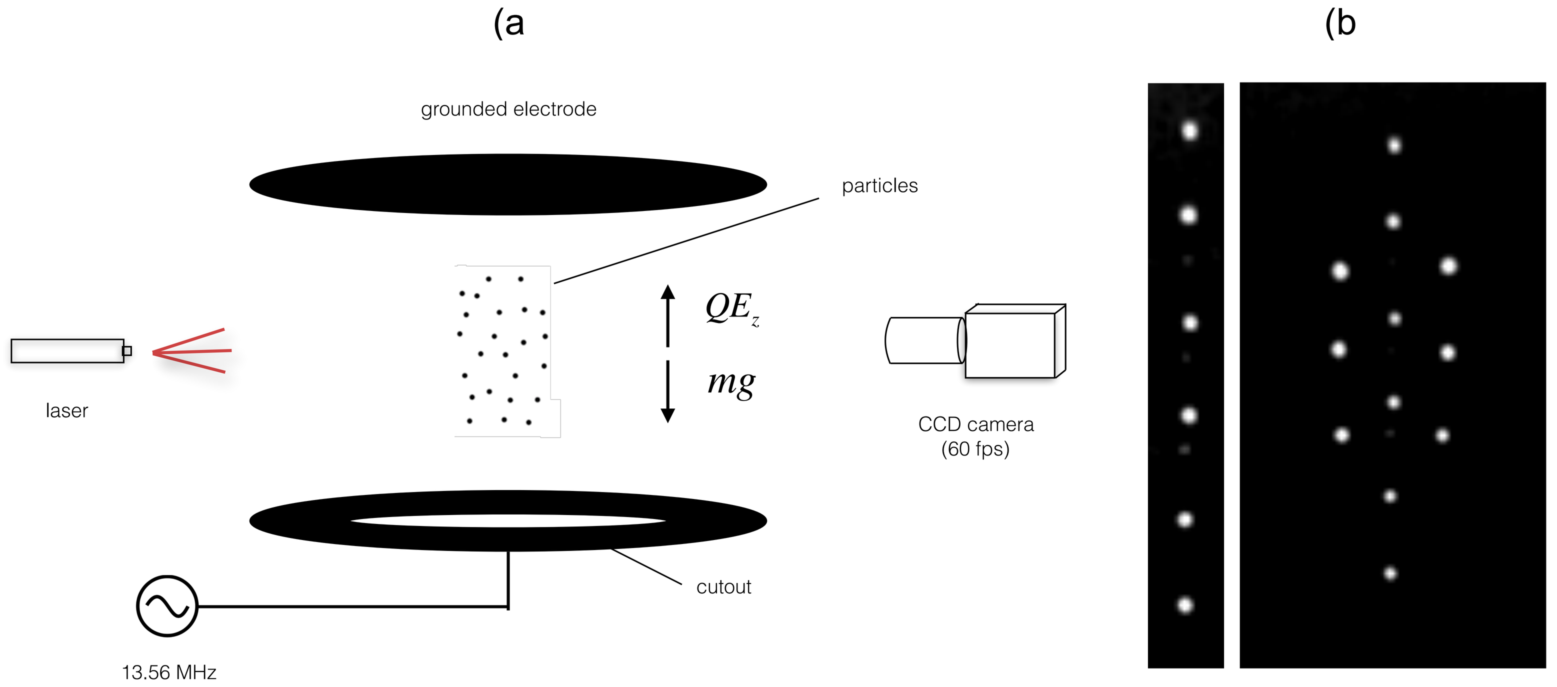


Figure 1. (a) Diagram of Gaseous Electronics Conference (GEC) reference cell in Baylor's Hypervelocity Impacts & Dusty Plasmas Lab. The lower electrode ionizes the Argon gas and is driven at 13.56 MHz. The power is 200 mV, and the pressure is 144 mTorr. Horizontal confinement is provided by a parabolic well due to the cutout and a glass box. Particles levitate in the sheath above the lower electrode, balanced by a self induced E field. Laser illumination allows CCD camera imaging at 60 fps. (b) Single and hexagonal chain configurations for 8.89 micrometer melamine formaldehyde particles. Hexagonal chain is single chain surrounded by six three-particle chains in the shape of a hexagon when viewed from above.

Methods

Normal Mode Analysis (NMA) was performed by recording the random Brownian motion of particles around their equilibrium positions $\vec{r}_i$ at 60 fps. The particle velocities $\vec{v}_i(t) = \Delta\vec{r}_i(t)/\Delta t$ were projected onto each mode eigenvector $\vec{a}_{i,\ell}$ with $\ell = 1,2,\ldots,3N$ representing mode number. Thus the time dependent signal

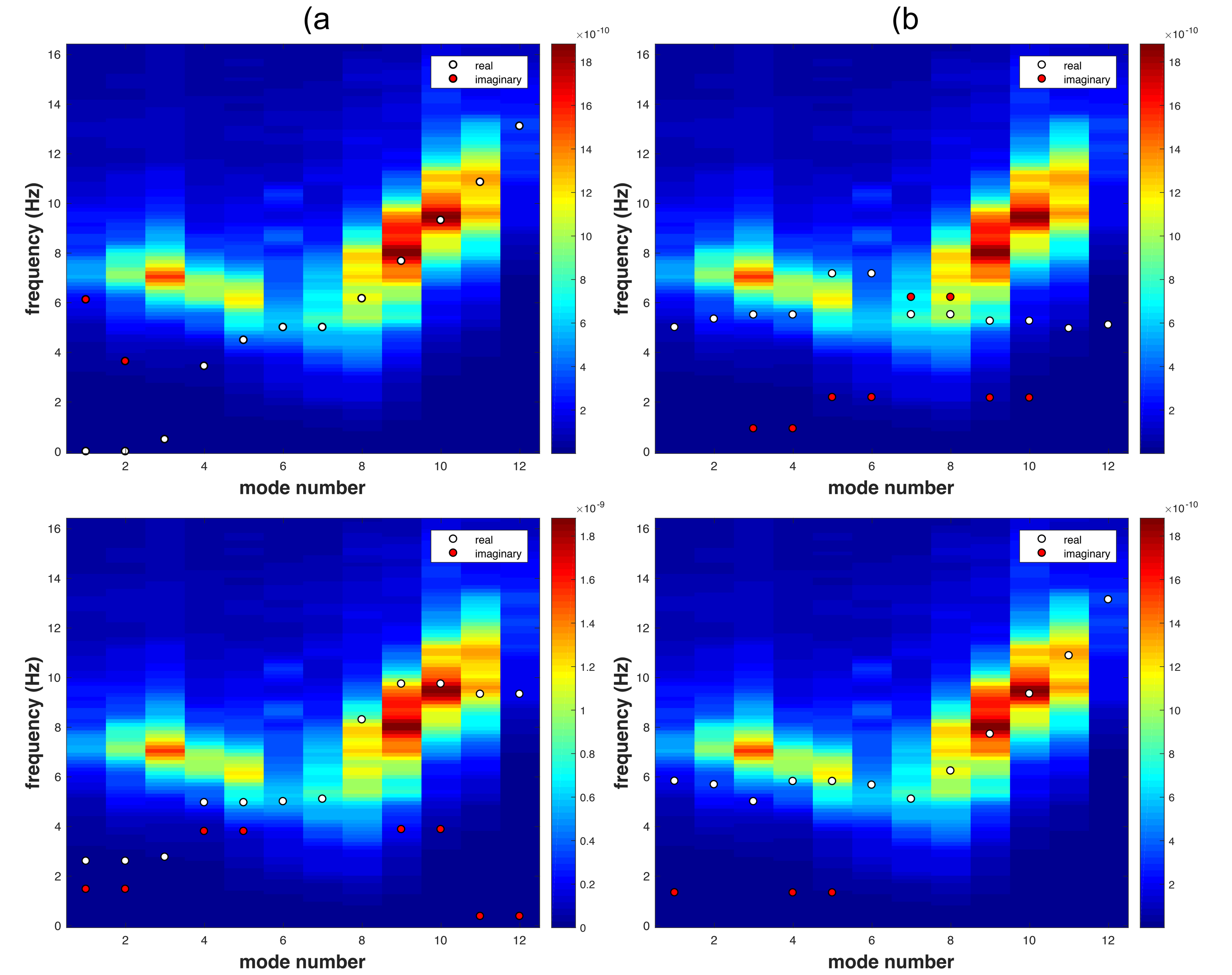
$$v_\ell(t) = \sum_{i=1}^{N} \vec{v}_i(t)\cdot\vec{a}_{i,\ell}$$

is calculated. Finally, the spectral power density [3] is defined to be

$$S_\ell(\omega) = \frac{2}{T}\left|\int_{-T/2}^{T/2} v_\ell(t)\cdot e^{-i\omega t}\,dt\right|^2$$

Results

(a (b



(c (d

Figure 2. The normal mode spectrum for a six particle, vertically aligned chain. The ionized Argon gas pressure = 144 mTorr, and the rf power = 200 mV. Modes 1-6 are horizontal oscillations; 7-12 are vertical. The color bar corresponds to measured spectral power density. White dots are the calculated eigenfrequencies based on a) Pure Yukawa potential, b) Yukawa + point charge potential, c) same as b, but only acting on downstream

It can be seen from Figure 1a that vertical motion is well described by a Yukawa potential, but horizontal motion is not. Imaginary frequencies correspond to absolutely instabilities in the system, as can be seen from an exponential growth or decay term in the equation of motion:

$$x(t) = Ae^{-i\omega t} = Ae^{-i(c+di)t} = Ae^{-ict}e^{dt}$$

The point charge model is a logical next step, in which an additional term is added to the Yukawa potential:

$$\phi(r) = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q}{r}e^{-r/\lambda_d} + \frac{q}{r_p}e^{-r_p/\lambda_d}\right]$$

where and art the charge and distance to the point charge. Figure 1b shows that this additional term disturbs the vertical modes, although leading to the semi-stable Schweigert instabilities in the horizontal modes (complex # frequencies). It can be concluded that the point charge model is inadequate for a vertically aligned structure; however, Figure 1c suggests a potential that only acts on downstream neighbors is more accurate.

A final analytical guess of the potential comes in the form of a harmonic oscillator potential well, decaying along the z axis:

$$\phi(\rho) = -\frac{1}{2}k\rho^2\left(e^{-z}\sin(z)\right)$$

where is the radial distance from the z axis. The sinusoidal term creates a positive space charge region, instead of a simple point charge. Figure 1d shows a semi-stable system, suggesting a wake force which is attractive mostly in the radial direction.

(a (b

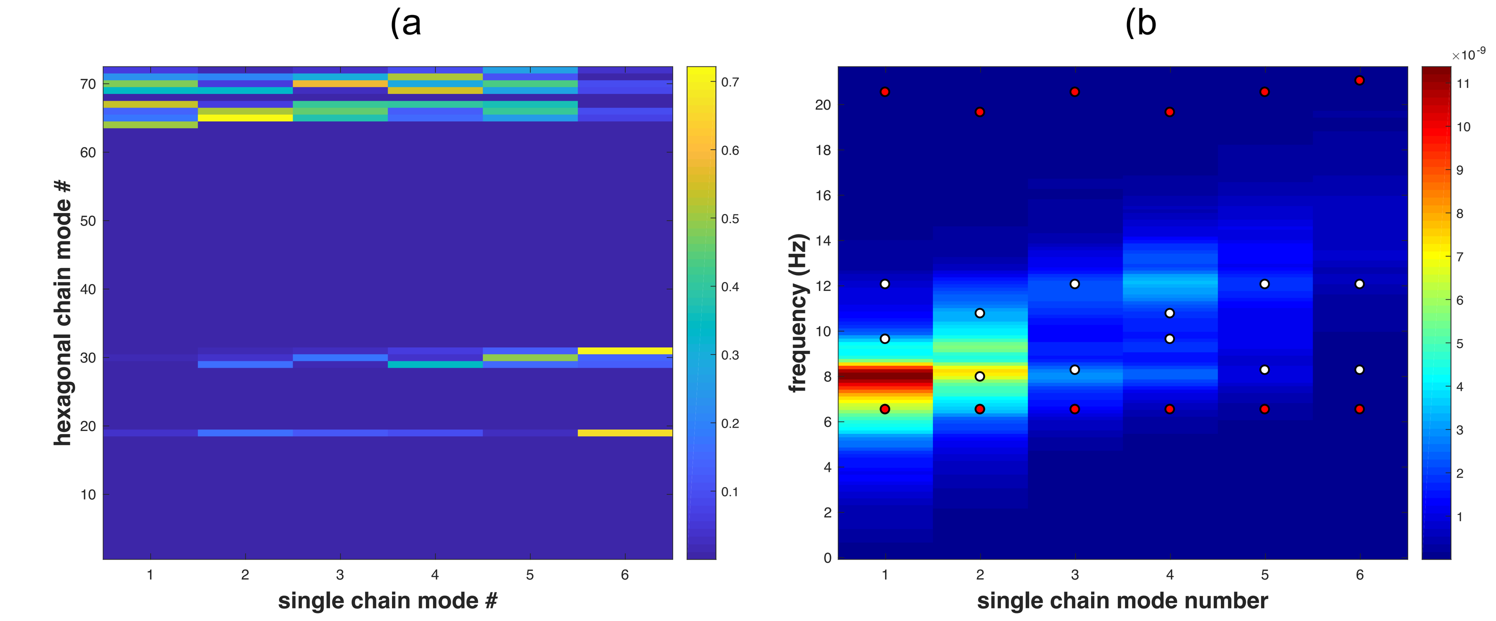


Figure 3. Relationship between vertical modes of a single chain and the middle chain of a hexagonal structure. High values correspond to strong correspondence between the modes. (b) Frequencies for the maximum of each apparent branch. The top two

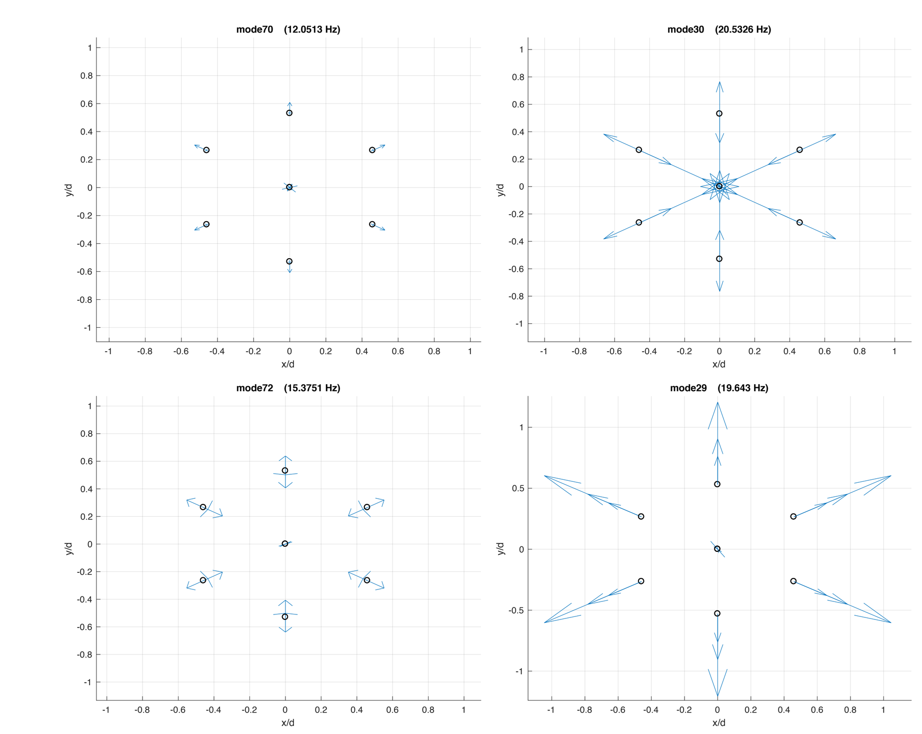


Figure 4. Radial direction eigenvectors for modes in top (left column) and bottom (right column) branches of Figure 3b. These modes produce frequencies which are higher/lower than experiment. The top of view of the hexagonal structure is shown: a vertical middle chain with 6 particles surrounded by 6 sub-chains of 3 particles each. Modes dominated by vertical sub-chain motion seem to agree more closely with experiment.

Figure 3 shows a double branching effect for the hexagonal structure mode spectrum, but a quadruple branch for the theoretical. Results are inconclusive. The top two branches (shown in white) present frequencies close to that of experiment, particularly modes 3 and 5. However, outlying frequencies exist at the bottom two branches. Observation of the mode eigenvectors in Figure 4 shows that the outlying branches are dominated by radial motion. Horizontal modes for the sub-chains may be coupled with vertical modes in the middle chain.

Conclusions

By fitting theoretical eigenfrequencies to a the normal mode spectrum for a single chain, a general description of the wake potential behind a particle was proposed. It is attractive in the radial direction, with a positive region around the position of the lower particle. It decays along the z axis, and it only acts on downstream particles. Additionally, the mode spectrum of the middle chain in a hexagonal structure suggests mode coupling between outer and inner chains.

Future work will seek to validate better models of the wake potential using NMA. Analytical expressions of the potential involve difficult complex integration, so a numerical implementation could studied using COPTIC, a particle-in-cell simulation. It is still unknown if downstream particles affect the wake of the top particle, which could have consequences in chains. Results could be generalized to a hexagonal structure.

References

[1] Ke Qiao, Jie Kong, Lorin S. Matthews, and Truell W. Hyde, Phys. Rev. E **91** 053101 (2015)
[2] A. Melzer, Phys. Rev. E **67**, 016411 (2003).
[3] H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher, and D. Mohlmann, Phys. Rev. Lett. 73, 652 (1994).

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What is a complex plasma?

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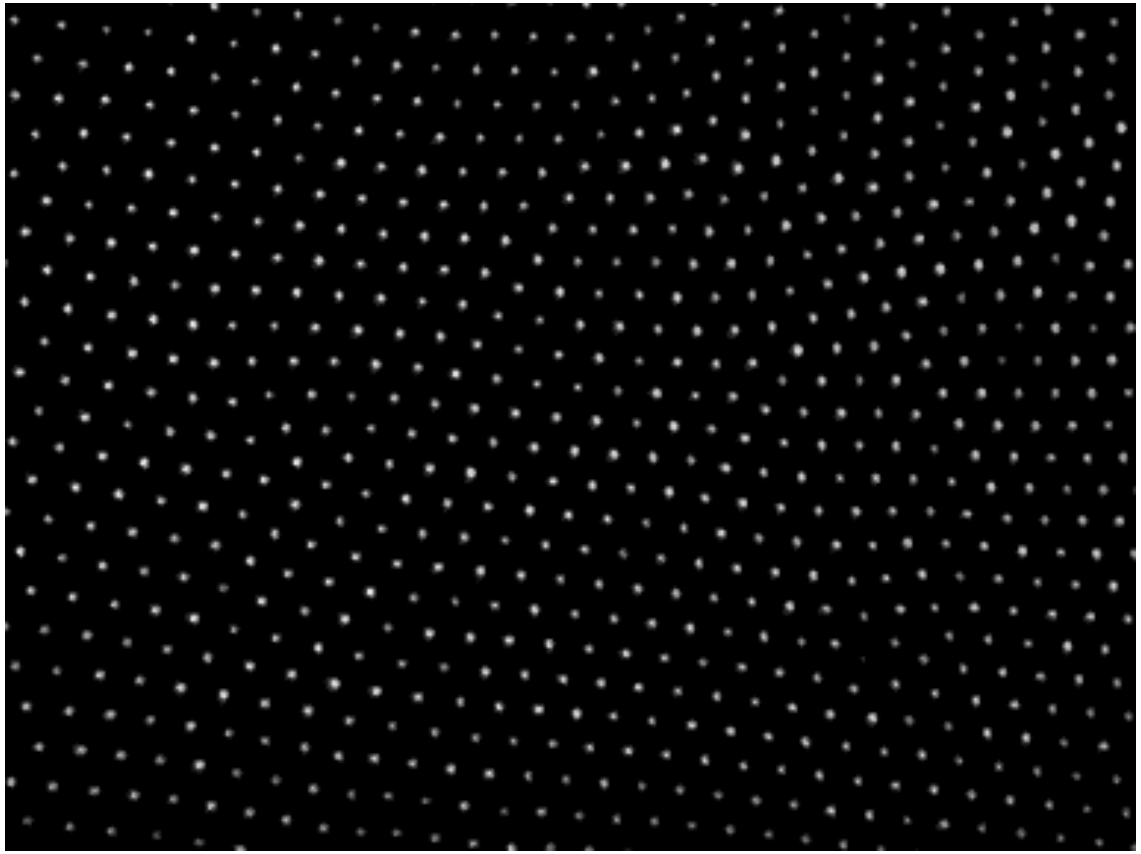
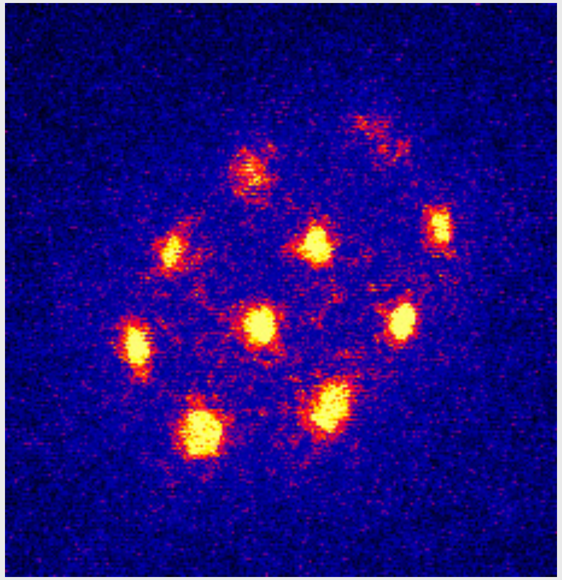


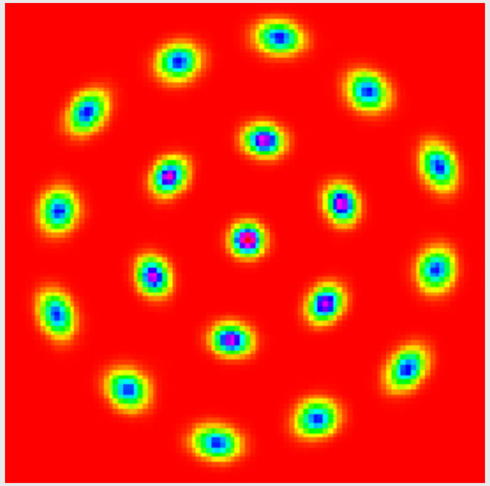
- Charged, strongly coupled micro-spheres in weakly ionized gas
- 1994 complex plasma crystal formed
- Open, self-organizing systems
- Length/ time scales ideal for video microscopy (10^{-6} m, 10^{-4} s)

Strongly correlated Coulomb systems in traps

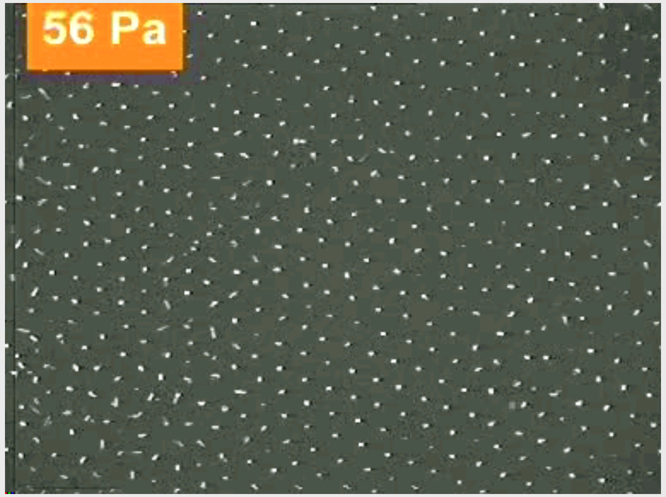


Ions in Paul-/Penning traps
G. Werth, Uni Mainz

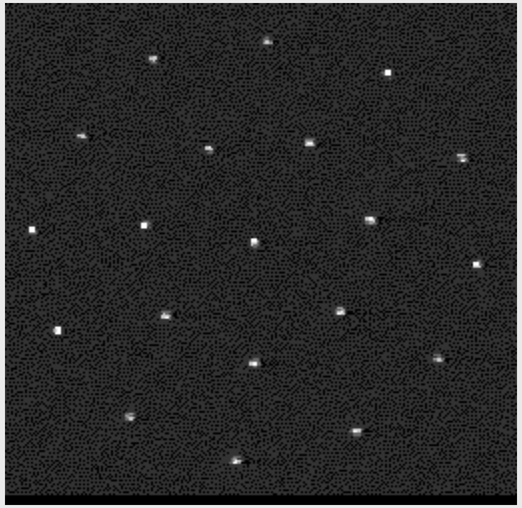
"Artificial Atoms"



Electron crystal
(Quantum dots)
A.Filinov, MB, Yu. Lozovik



Dusty Plasmas, Coulomb crystal
A. Piel, A. Melzer



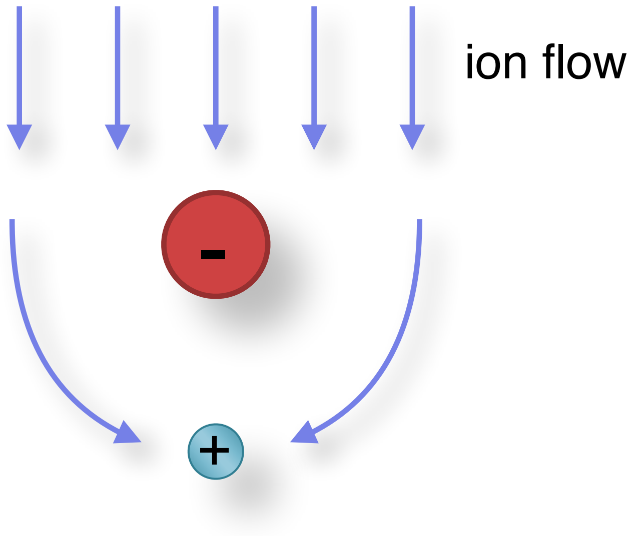
*Slide taken from: *Ladungen en der Falle* ©Michael Bonitz, Universität Kiel, 2006

Wake Potential

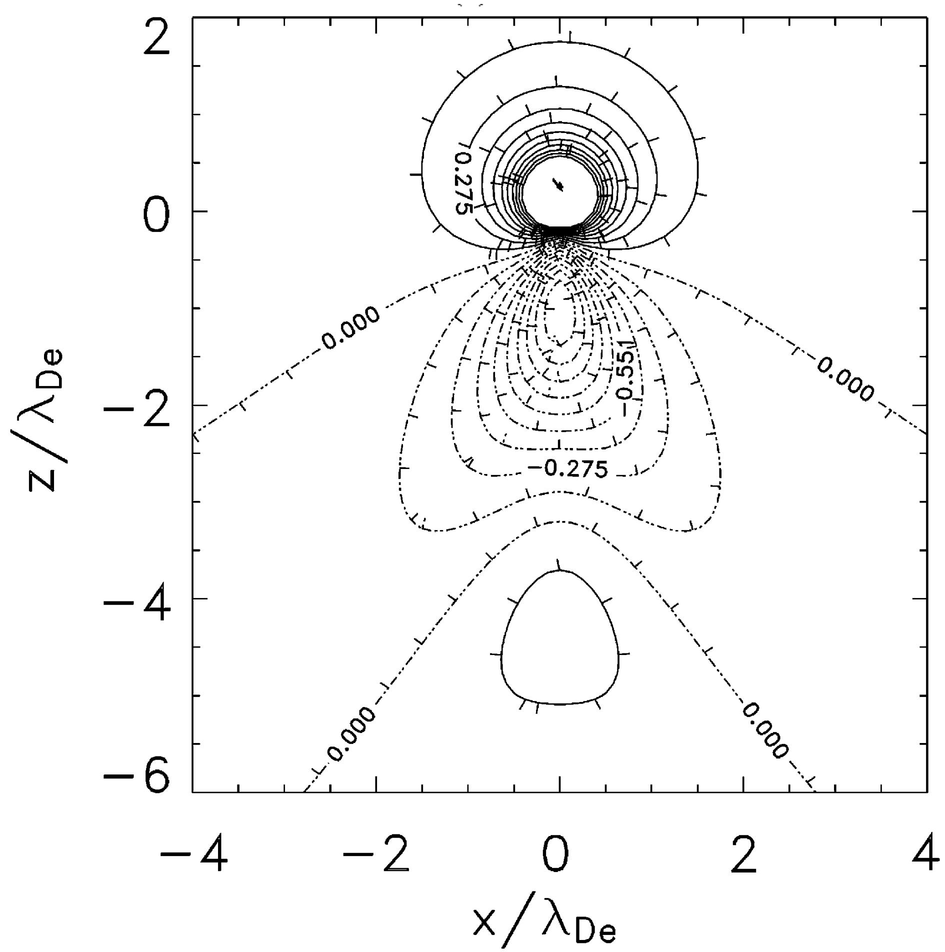
$$\phi(r) = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}e^{-r/\lambda_D}$$

$$\lambda_D = \sqrt{\frac{T\varepsilon_o k}{n_e e^2}}$$

Point Charge Model



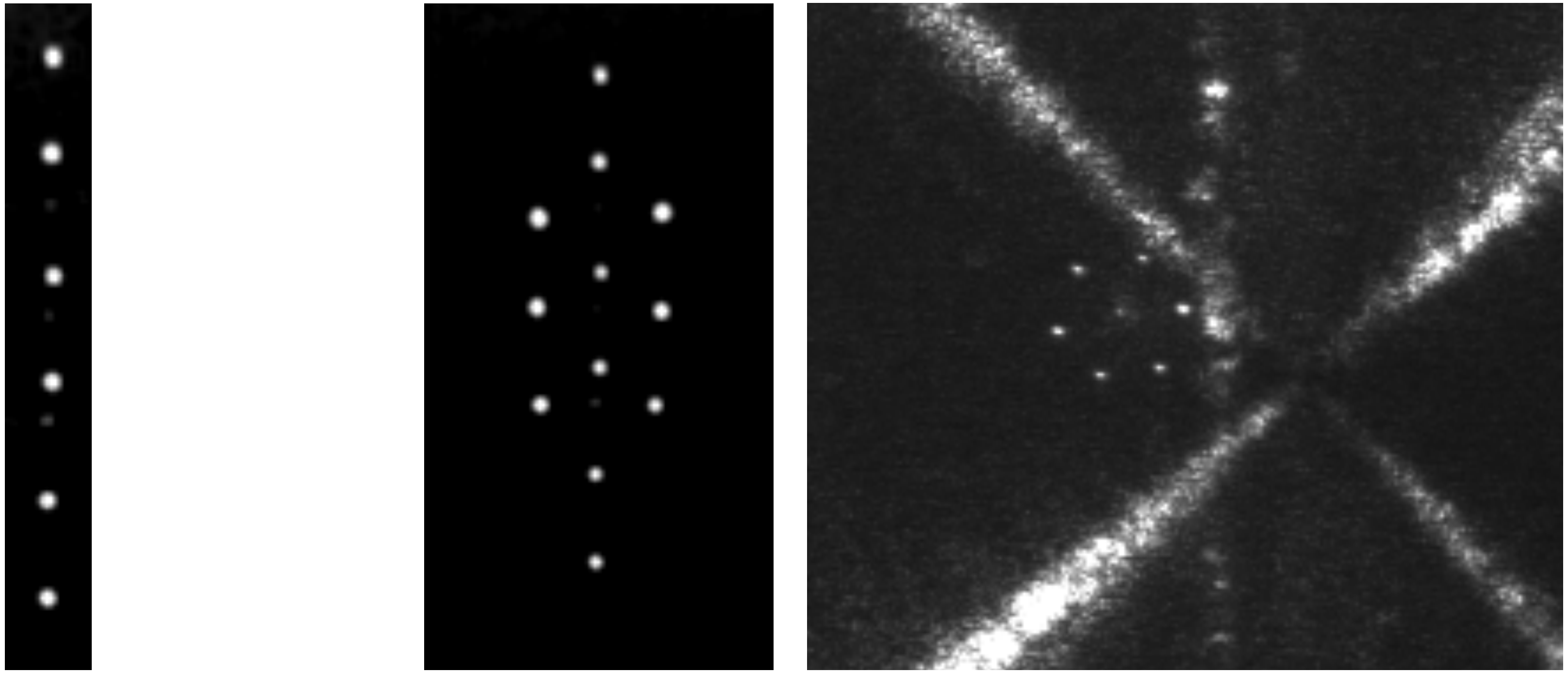
Linear Response Theory



Lampe

Project Goals

- Propose general description of wake potential by using NMA on single chain
- Search for evidence of mode coupling between chains in multi-chain dust structure

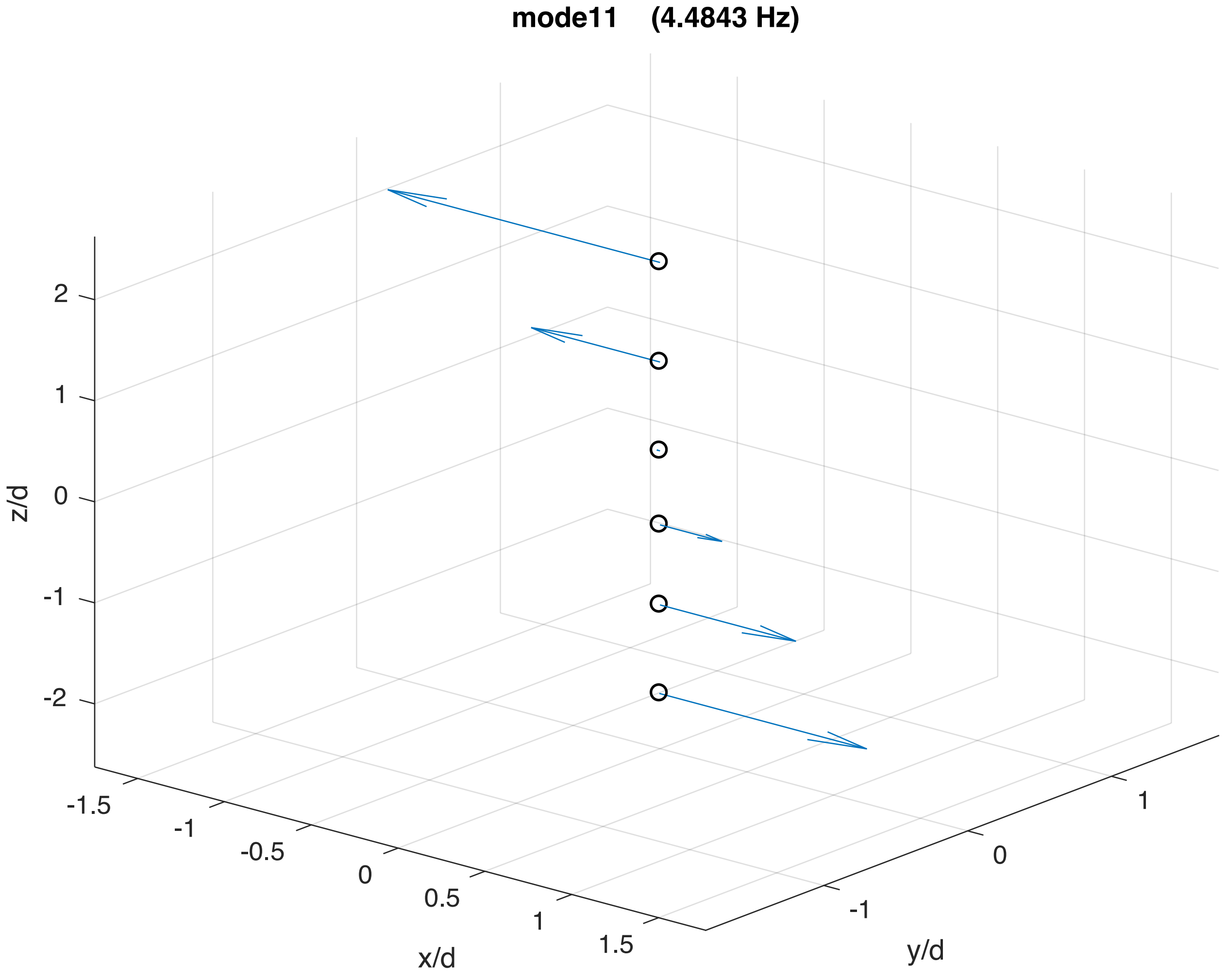


Normal Mode Analysis (NMA)

*mode = all components move at one same frequency

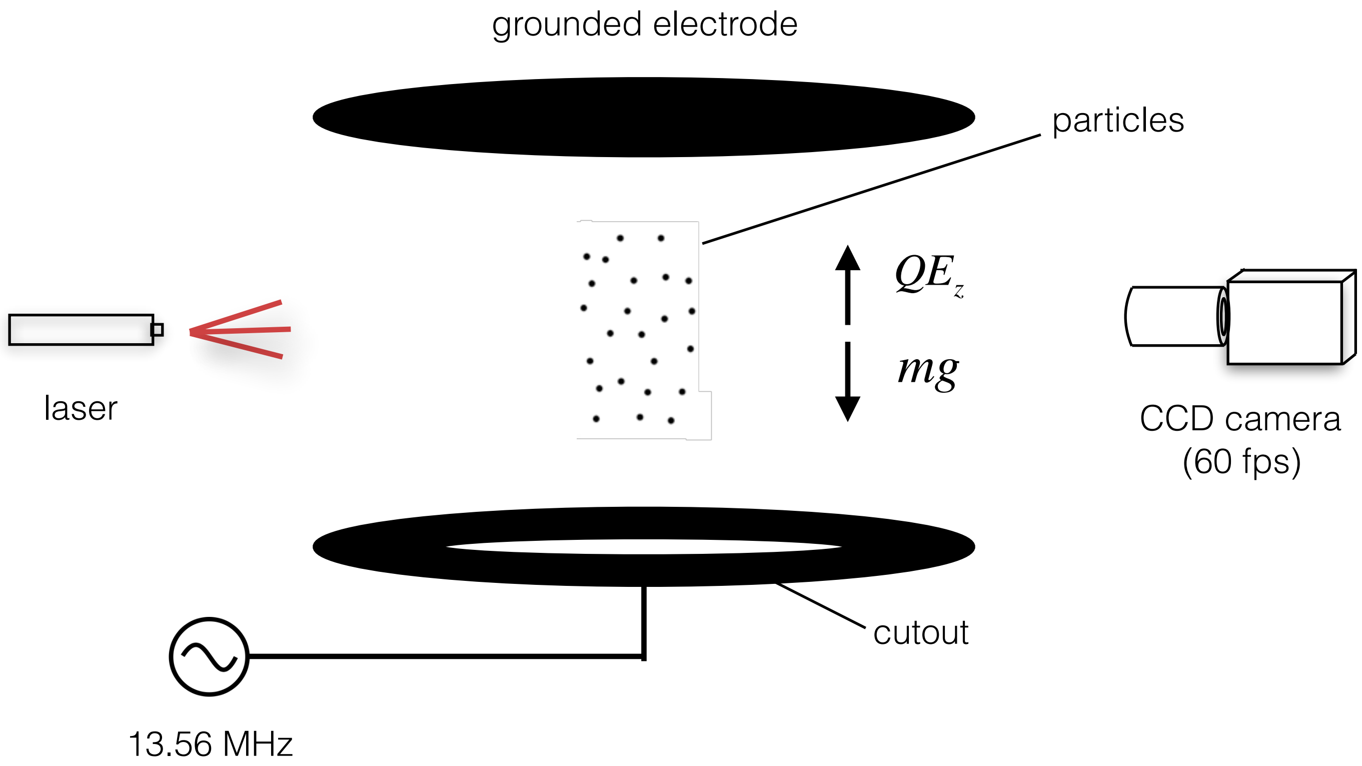
$$M\ddot{\vec{r}} = D_{\alpha i,\beta j}\vec{r}$$

$$D_{\alpha i,\beta j} = \left(\frac{\partial F_{\alpha i}}{\partial r_{\beta j}}\right)$$



Experimental Methods

- Gaseous Electronics Conference rf reference cell in Baylor's Hypervelocity Impacts & Dusty Plasmas Lab

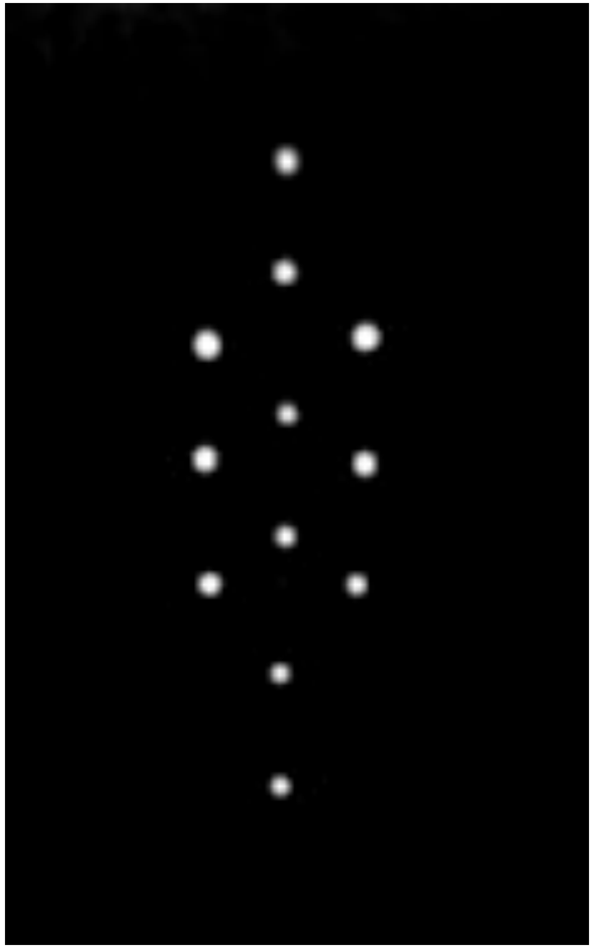


- Power = 200 mV
- Pressure = 140 mTor

Video Microscopy

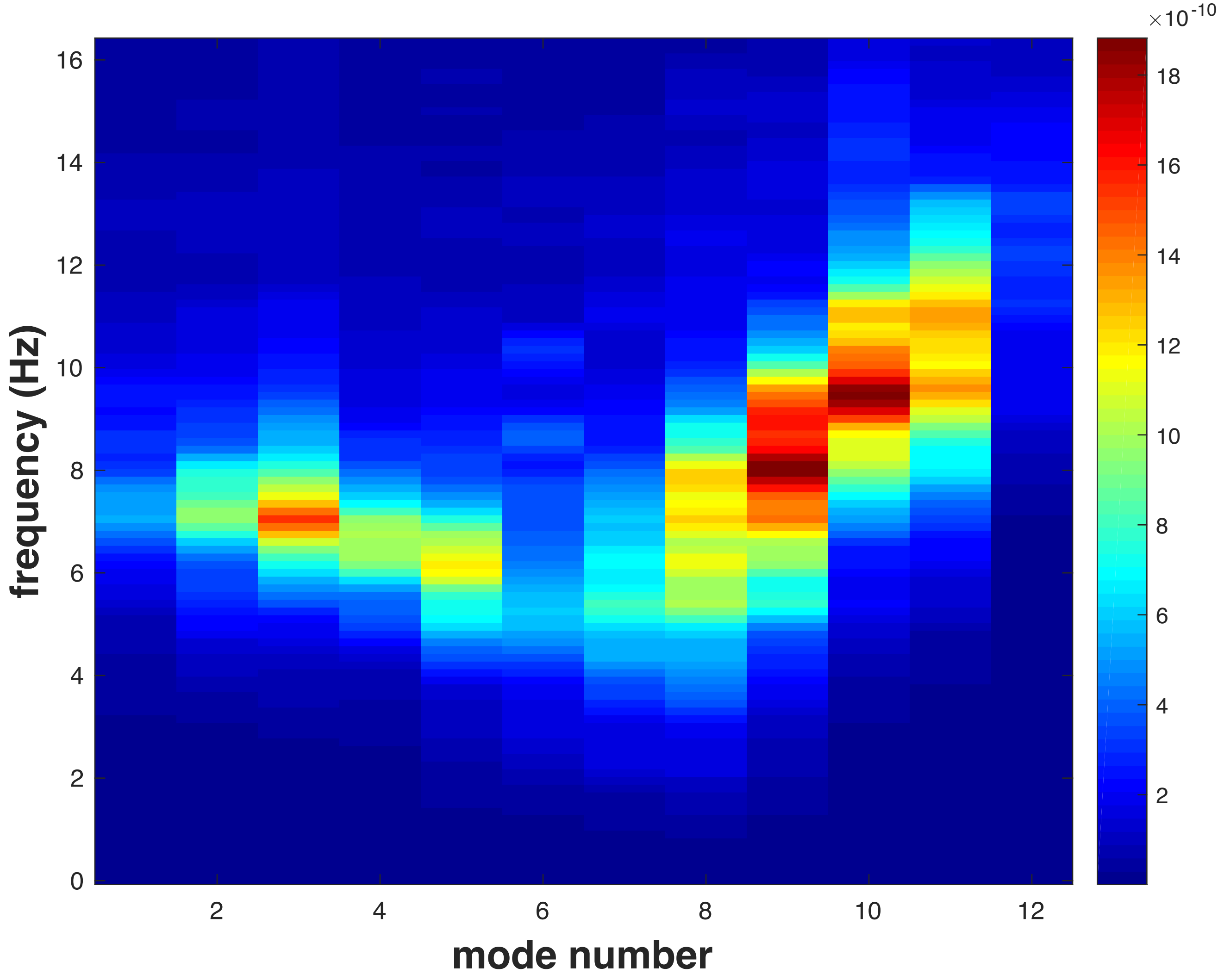
- Observe random thermal motion of particles $$\vec{v}_i(t) = \Delta\vec{r}_i(t)/\Delta t$$
- Project motion onto each mode eigenvector $(\ell = 1,2,\ldots,3N)$ $$v_\ell(t) = \sum_{i=1}^{N} \vec{v}_i(t)\cdot\vec{a}_{i,\ell}$$
- Spectral Power Density $$S_\ell(\omega) = \frac{2}{T}\left|\int_{-T/2}^{T/2} v_\ell(t)\cdot e^{-i\omega t}\,dt\right|^2$$

N = 4



N = 24 (12 visible)

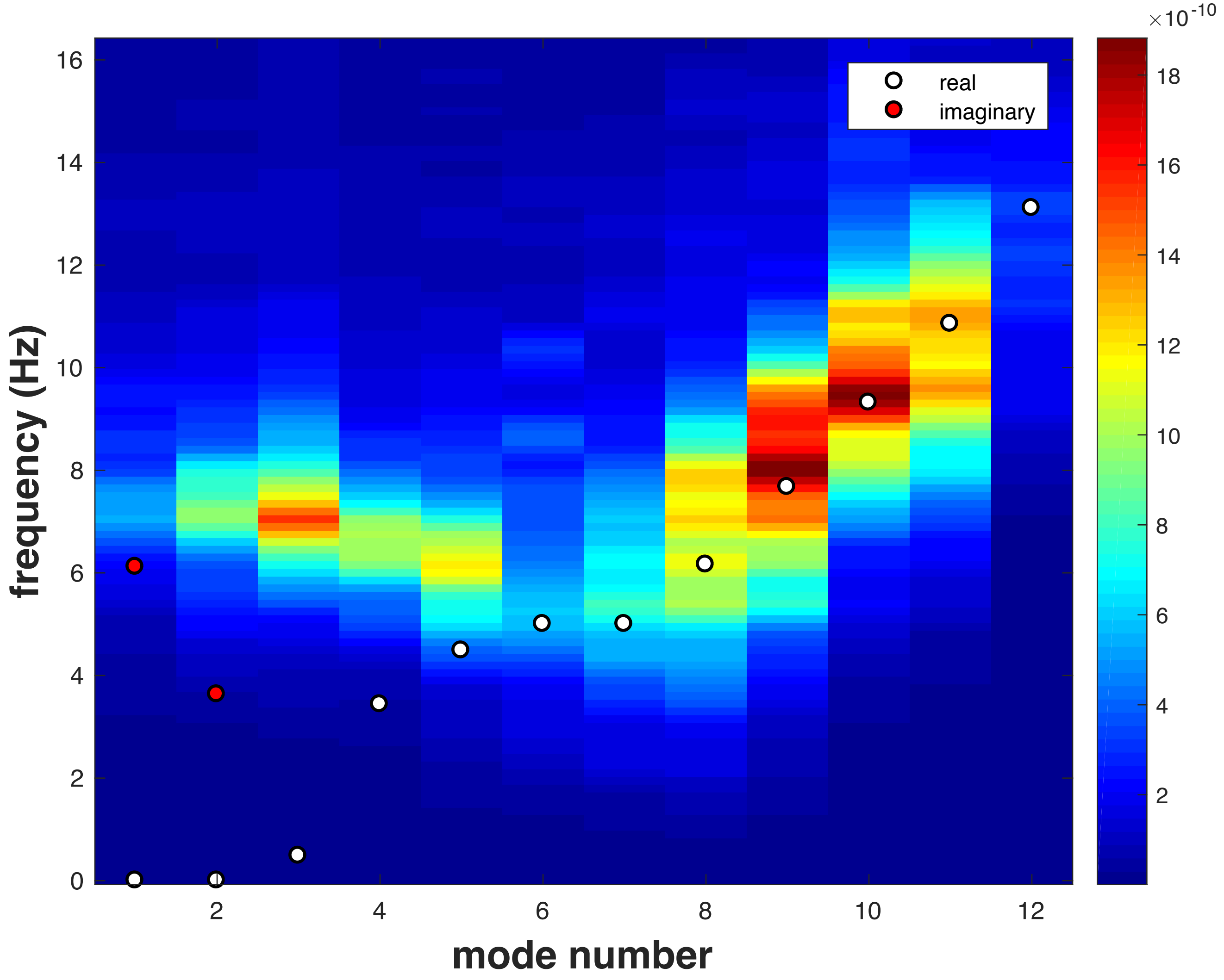
Single Chain Mode Spectrum



- Modes 1-6 = horizontal
- Modes 7-12 = vertical

Color bar = spectral power density (SPD)

Yukawa Model

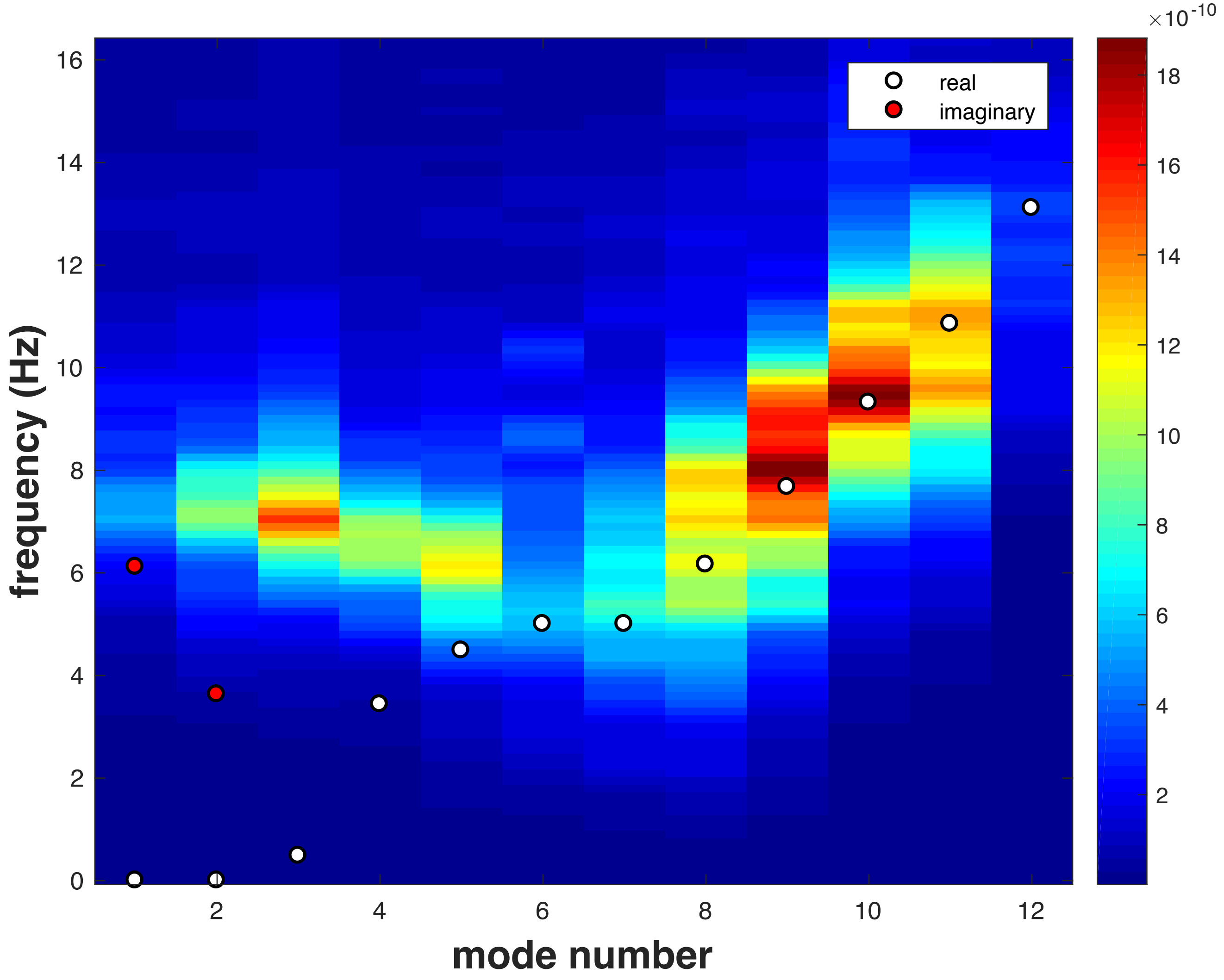


<u>Theoretical Mode Frequencies</u>

- Modes 1-6 = horizontal
- Modes 7-12 = vertical

$$\phi(r) = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}e^{-r/\lambda_D}$$

Yukawa Model



$$\omega_s = a + bi$$

$$x(t) = e^{-i\omega_s t} = e^{-iat} e^{bt}$$

System states

1. Stable (real frequencies)
2. Absolute Unstable (imaginary)
3. Schweigert Unstable (complex)

Point Charge Model

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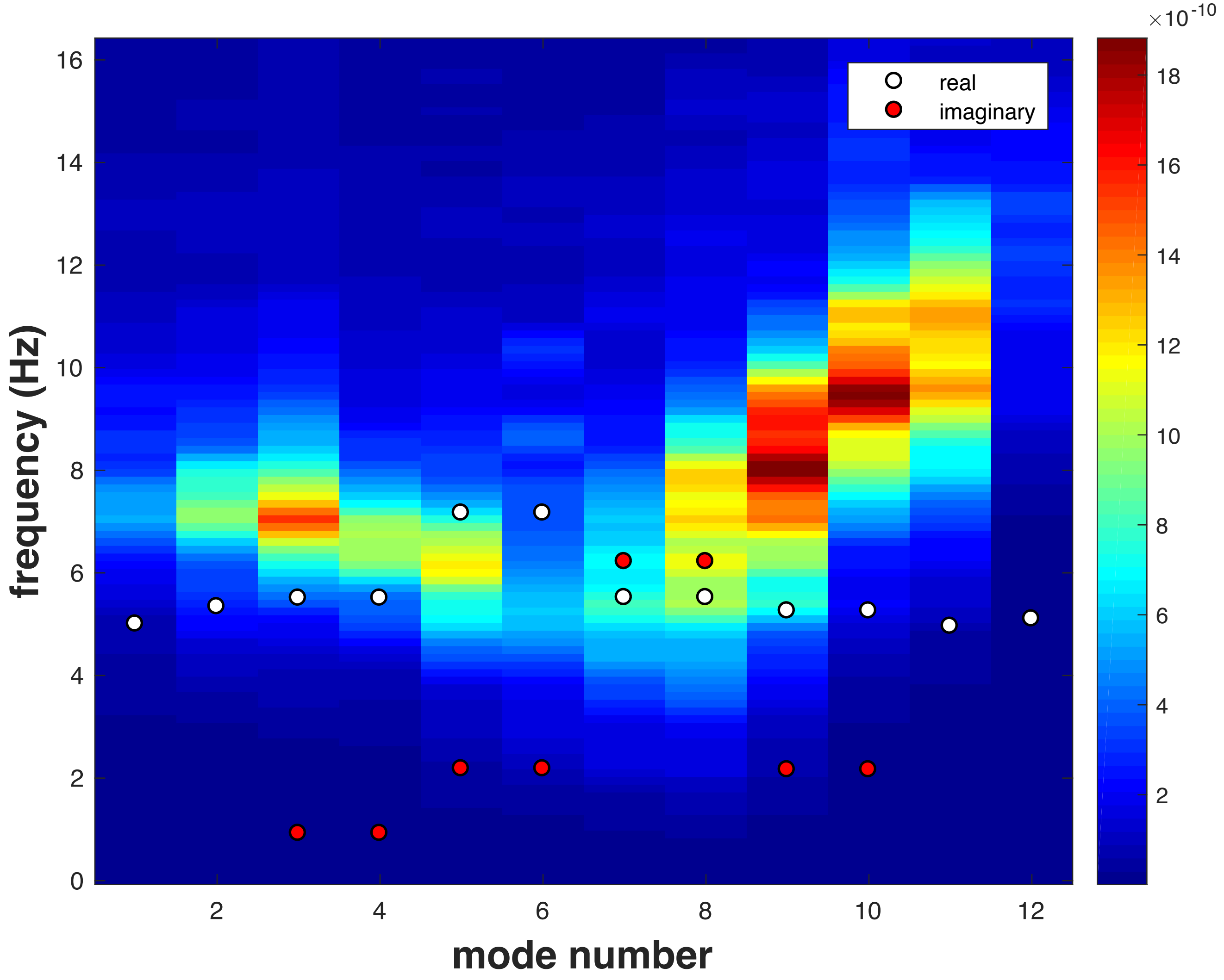
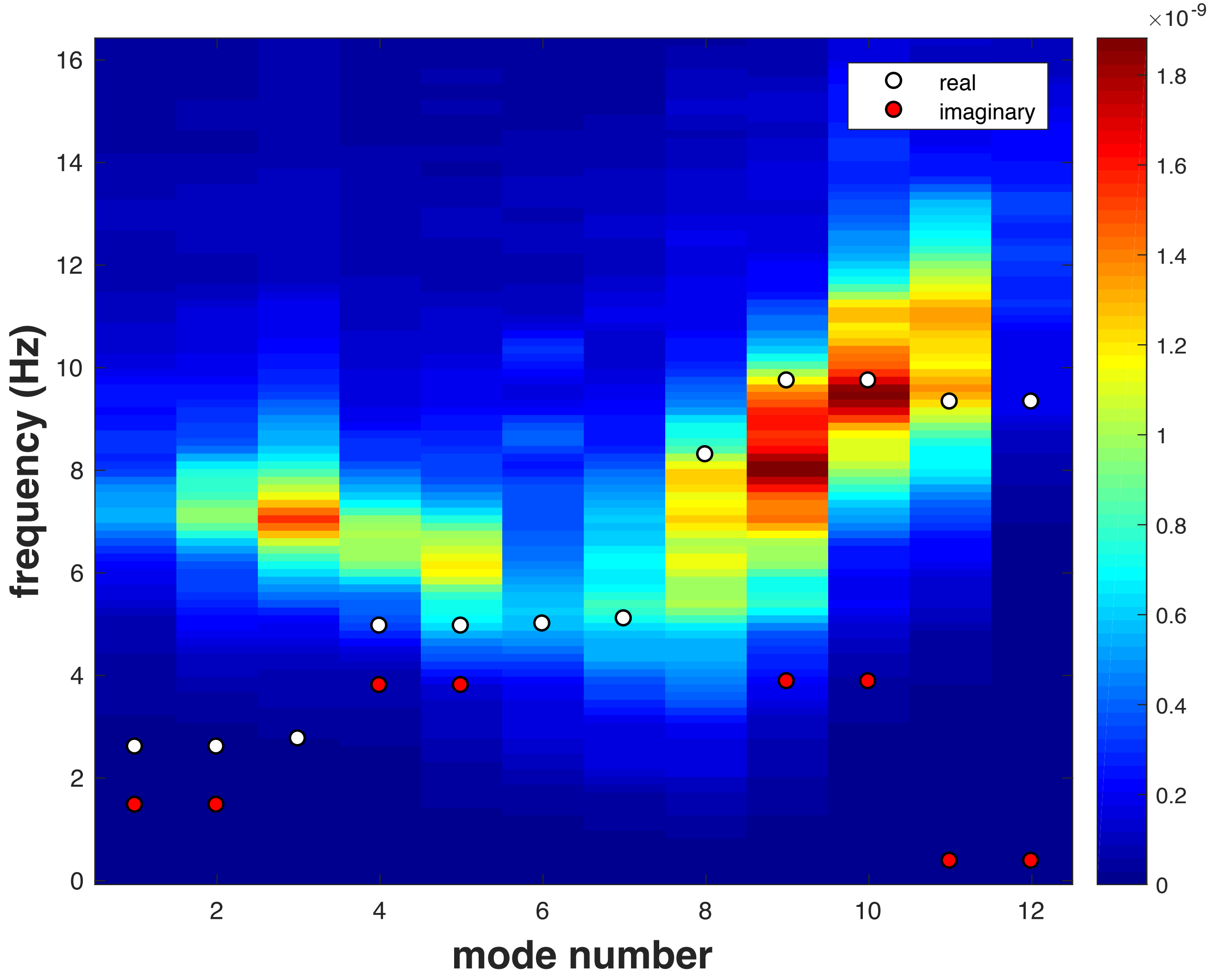


$$\phi(r)=\frac{1}{4\pi\varepsilon_o}\left[\frac{Q}{r}e^{-r/\lambda_d}+\frac{q}{r_p}e^{-r_p/\lambda_d}\right]$$

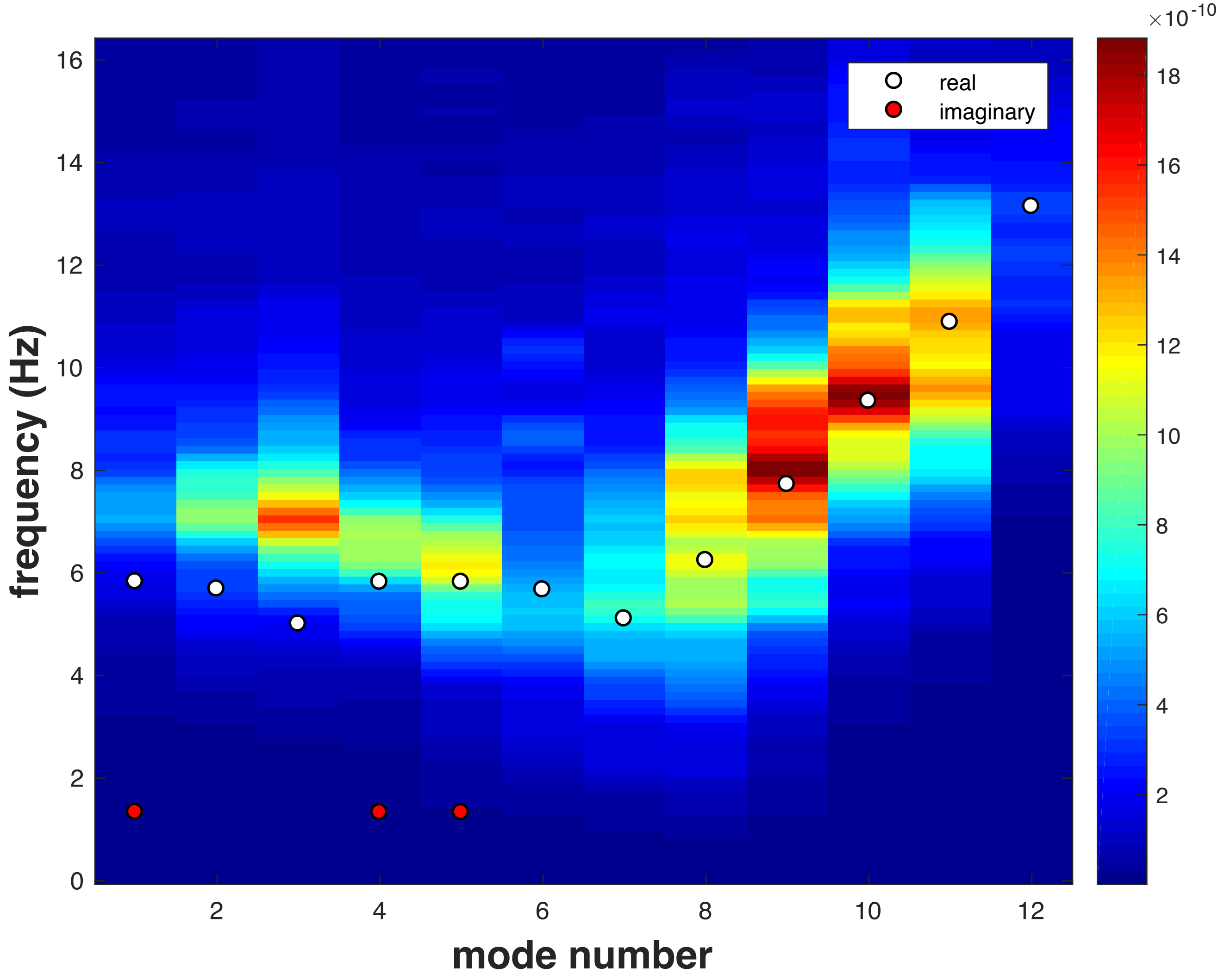
Point Charge Model



$$\phi(r)=\frac{1}{4\pi\varepsilon_o}\left[\frac{Q}{r}e^{-r/\lambda_d}+\frac{q}{r_p}e^{-r_p/\lambda_d}\right]$$

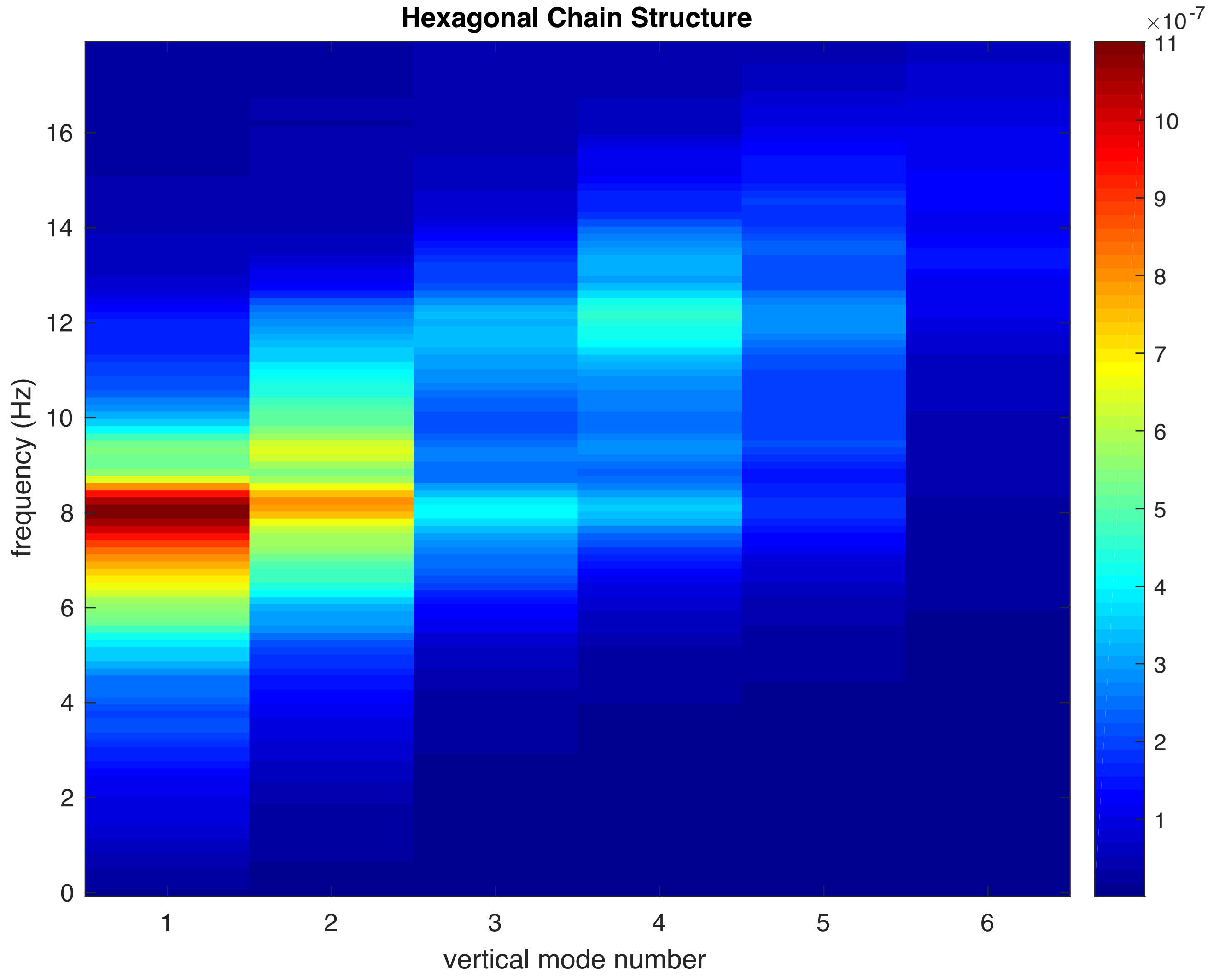
- Upstream = Yukawa, Downstream = Yukawa + point charge

Decaying, oscillating potential



$$F_\rho = -k\rho\left(e^{-z}\sin(z)\right)$$

HCS - Middle Chain Mode Spectrum



HCS - Middle Chain Mode Spectrum

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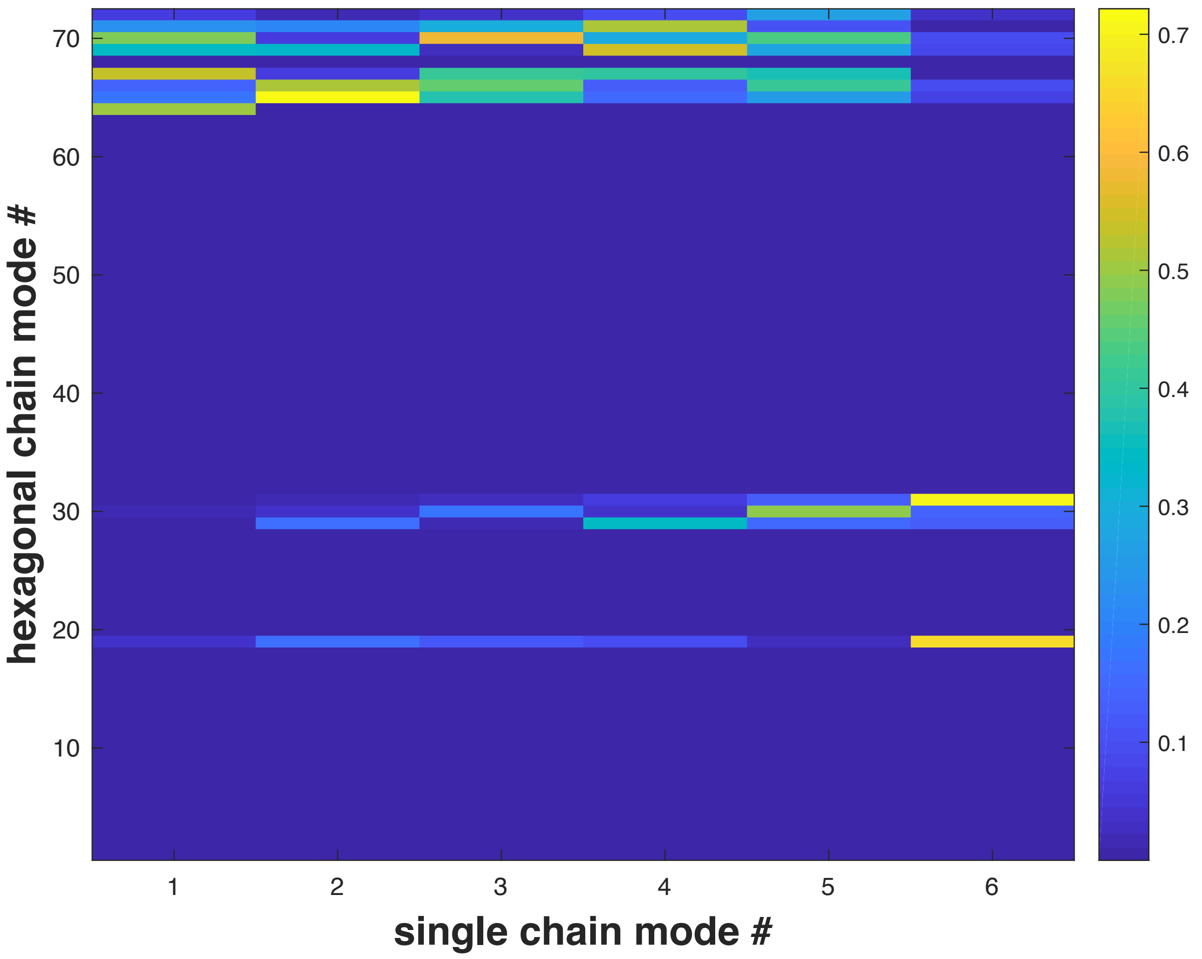

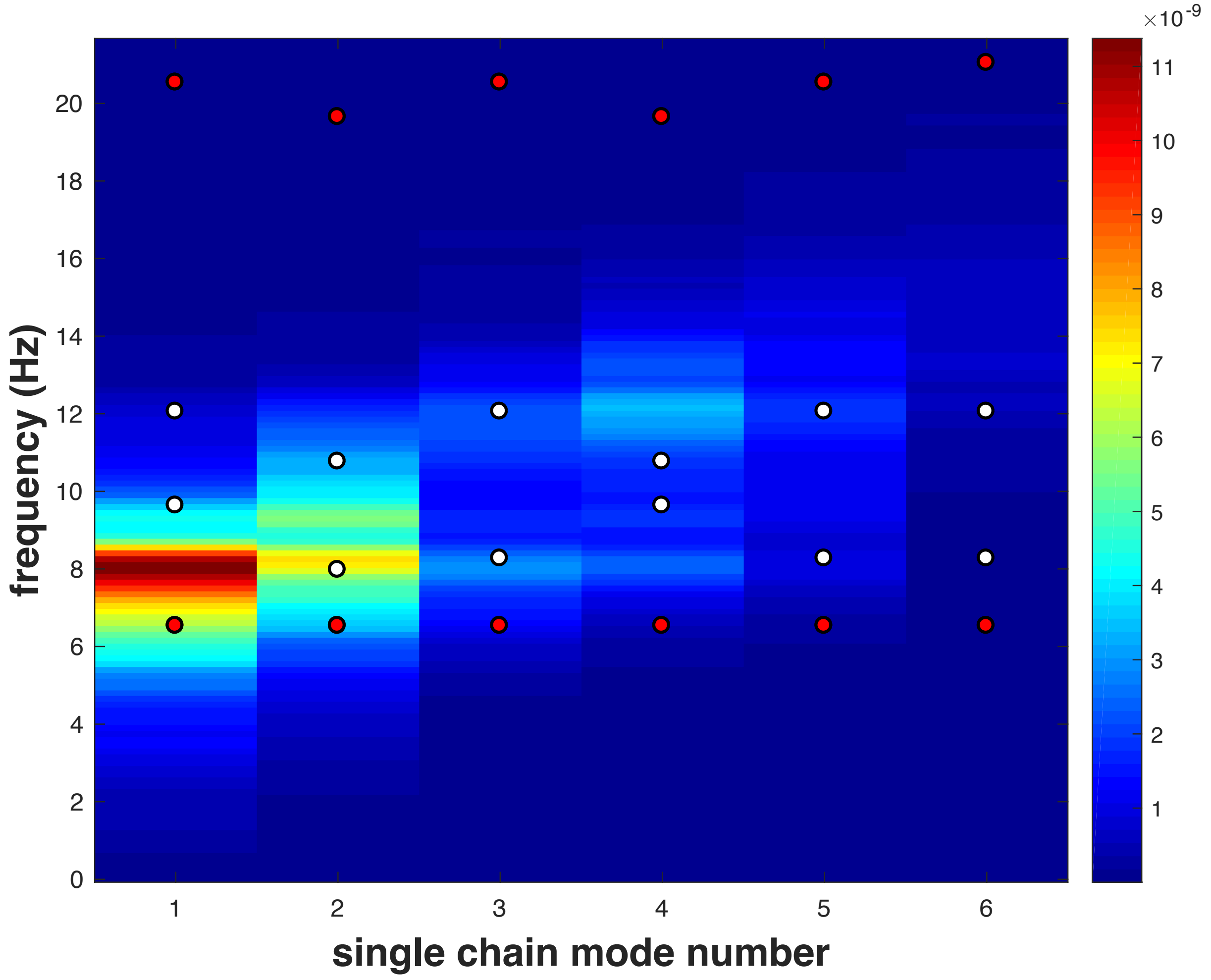

HCS - Middle Chain Mode Spectrum

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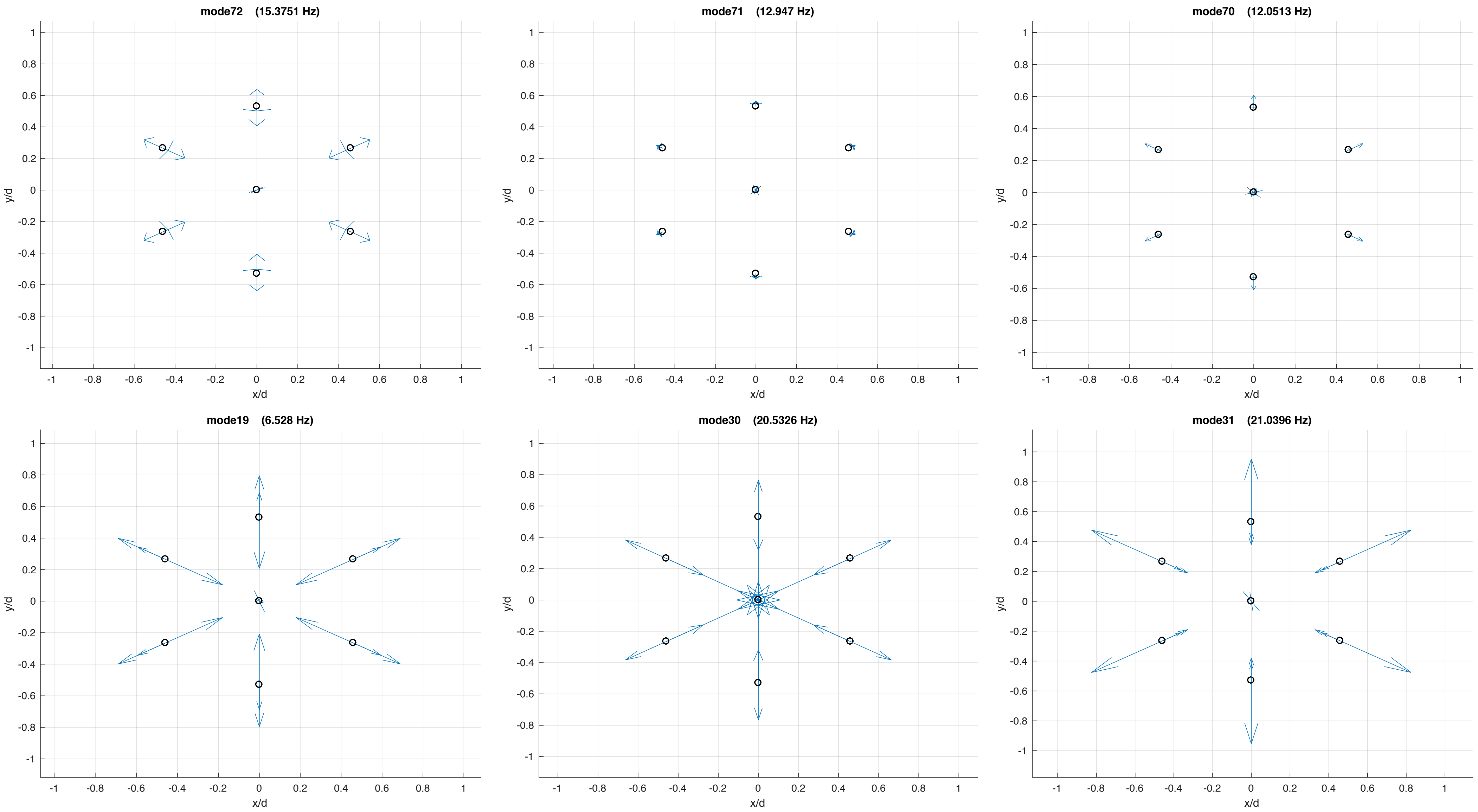

Conclusions

- Existence of an attractive downstream wake force was seen using NMA for a single chain.
- Data suggests mode coupling between chains in the hexagonal chain structure.

Future work

- Find mode eigenvalues numerically using COPTIC - PIC simulation

- Double branching mode spectrum is still unexplained

Thanks CASPER!