# Spectral Approach to Searching for Extended States in 2D Plasma Crystals

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## Anderson Localization

- In the tight Binding model with disorder, extended states may become localized by self interference from phase changed parts of the wavefunction reflected from crystal defects. This is the point of MIT transition
- For classical waves, MIT transition occurs when mean free path length between scattering events is comparable to the wavelength.
- Localization: wave amplitude falling off as an exponential in space. Delocalization for a classical wave is defined as energy dropping off nearly linearly. This corresponds to Ohm's law.
- The classical solution to the anderson type hamiltonian for particles is Einsteinian brownian motion - always diffusion for nonzero D

$$\vec{J} = -D\nabla\phi$$

- Variation of onsite energies and hopping potential has the same effect as variation of one of these variables - We vary onsite energy to get the hamiltonian of the form[1],

$$\hat{H} = \begin{pmatrix} \epsilon_1 & I & 0 & 0 \\ I & \epsilon_2 & I & 0 \\ 0 & I & \epsilon_3 & I \\ 0 & 0 & I & \epsilon_4 \end{pmatrix} \qquad \hat{H}_\epsilon = -\Delta + \sum_{i\in\Gamma} \epsilon_i \delta_i |\delta_i\rangle$$

where I is the hopping potential, $\delta_i$ is 1 for ith onsite energy and 0 otherwise, and $\Delta$ is the laplacian. $\epsilon_i$ are i.i.d. distributed random variables on the interval [-W/2,W/2]. W is the width of energy distribution.

## Anderson Localization and the Spectral Method

- Spectral theory is a generalization of eigendecomposition based on the ability to decompose operators into outer products of "cobasis vectors".
- On a continuous basis, spectral theory allows for a change of measure in the completeness relation to include a weight function multiplying an eigenvalue function, on the discrete and absolutely continuous parts [2]:

$$\int f\,d\mu = \int \left[f[\delta(\xi-\lambda_1)+\delta(\xi-\lambda_2)+\cdots]+w(\xi)f(\xi)\right]d\xi$$

- There is another "discrete-continuous" part of the measure that cannot be easily written down.
- For Schrodinger's equation it is clear that the continuous part corresponds to extended states, and the discrete part to localized states.
- Vectors corresponding to the continuous part of the measure are "cyclic" in the hamiltonian. i.e a state $|\psi>$ is an extended state if: $span\left\{|\psi\rangle, \hat{H}|\psi\rangle, \hat{H}^2|\psi\rangle, \cdots\right\} = \mathbb{R}^2$ in 2D[1]. This span is called the cyclic subspace.
- This makes sense - hamiltonian is the generator of time translation and in time evolving an extended state it will spread out in space like a gaussian wave packet. Here H has a continuous eigenspectra.
- Scaling theory solves the hamiltonian numerically with application of different boundary conditions(2016 Nobel Prize).
- Scaling theory appears to fail for 2D where it always predicts localization if there's disorder. Mathematicians say it bounds the Hilbert space and may throw out some solutions.

### Numerical experiment in Complex Plasma Crystals

- The fact scaling theory appears to fail in the 2D case is very relevant to 2D materials such as graphene.
- It is simple to create honeycomb lattice plasma crystal graphene analogues using PK-4 mechanism[4].
- PK-4 creates a plasma using RF frequency waves to ionize gas, and plastic spheres are introduced into the gas. They charge, and a radial confinement E-field is applied by holding a parabolic indentation below the crystal at a certain potential, electrodes balance gravity.
- Crystal generation is modeled using the box_tree code developed by Derek Richardson and Lorin Matthews, with variation in confinements and particles sizes.
- A number of crystals at equilibrium with different disorders were generated.
- Disorder is measured by the "number of particles with a different number of nearest neighbours/the number of particles with 6 nearest neighbors" - called the defect in solid state.

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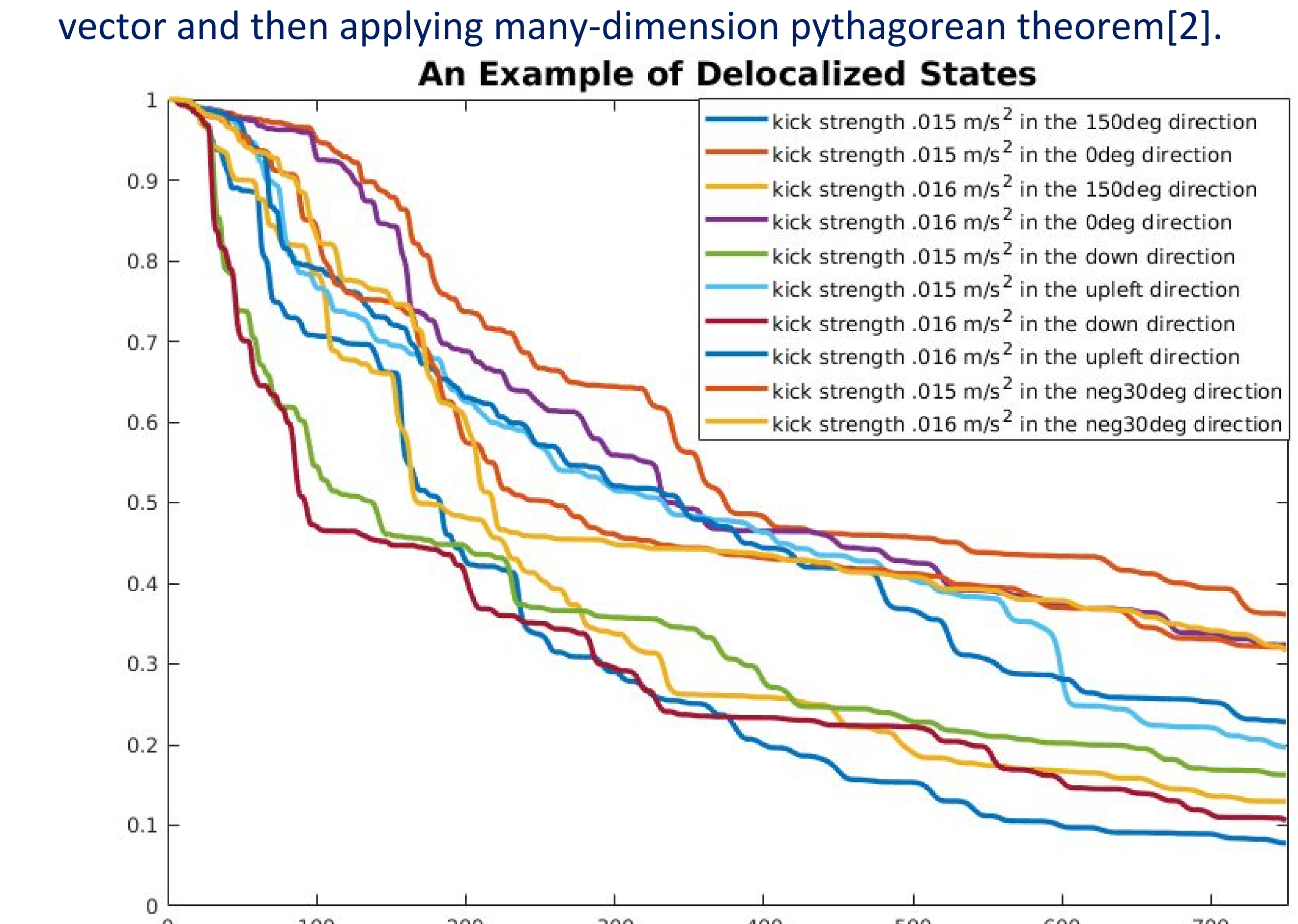
- The particle nearest the center was perturbed with a gaussian force packet applied to it for .04 seconds in a number of different directions and contributed to the acceleration of the particles with .014,.015,.016,.017,.018 and .019 m/s$^2$.
- The positions and velocities for all the particles were recorded for a few seconds after the kick is applied. These data are used to check localization by calculating the distance to the cyclic subspace in each timestep. The distance to the subspace is found using a combination of Grahm-Schmidt type sum of projections of all vectors normal to the cyclic vector and then applying many-dimension pythagorean theorem[2].




Plot of distance to the cyclic subspace for delocalized state. Although no delocalized kicks at equilibrium were finished by the time to print this poster, this is an illustration of states that are delocalized because the crystal is expanding(not at equilibrium).

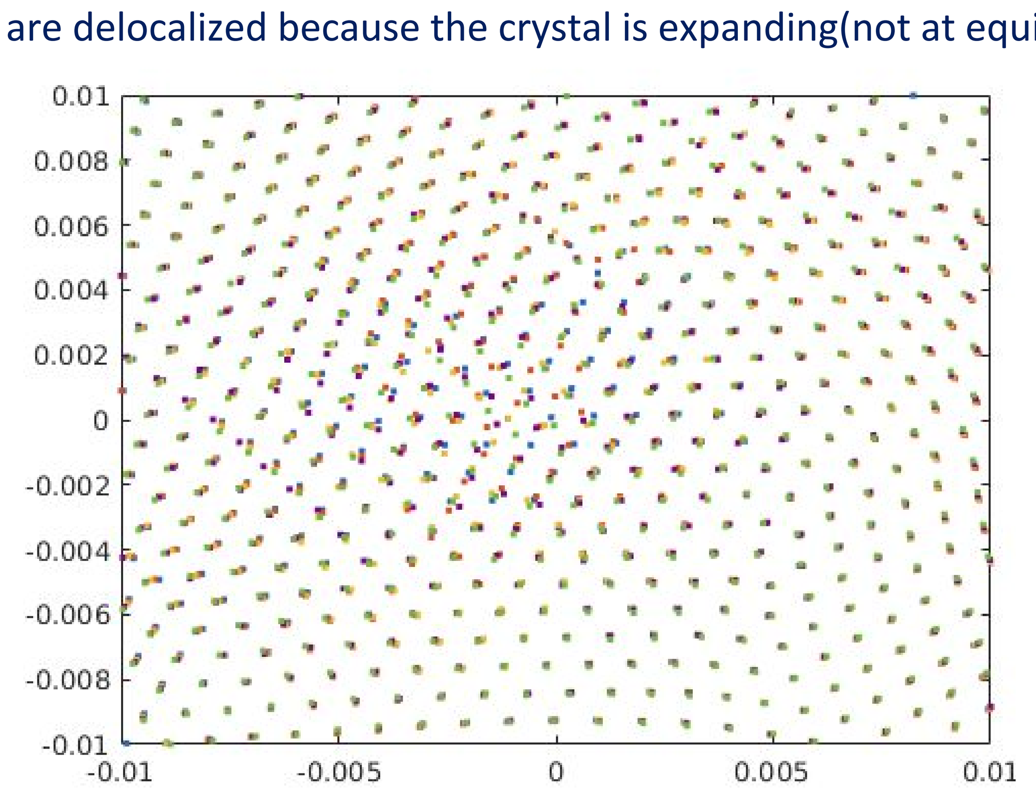
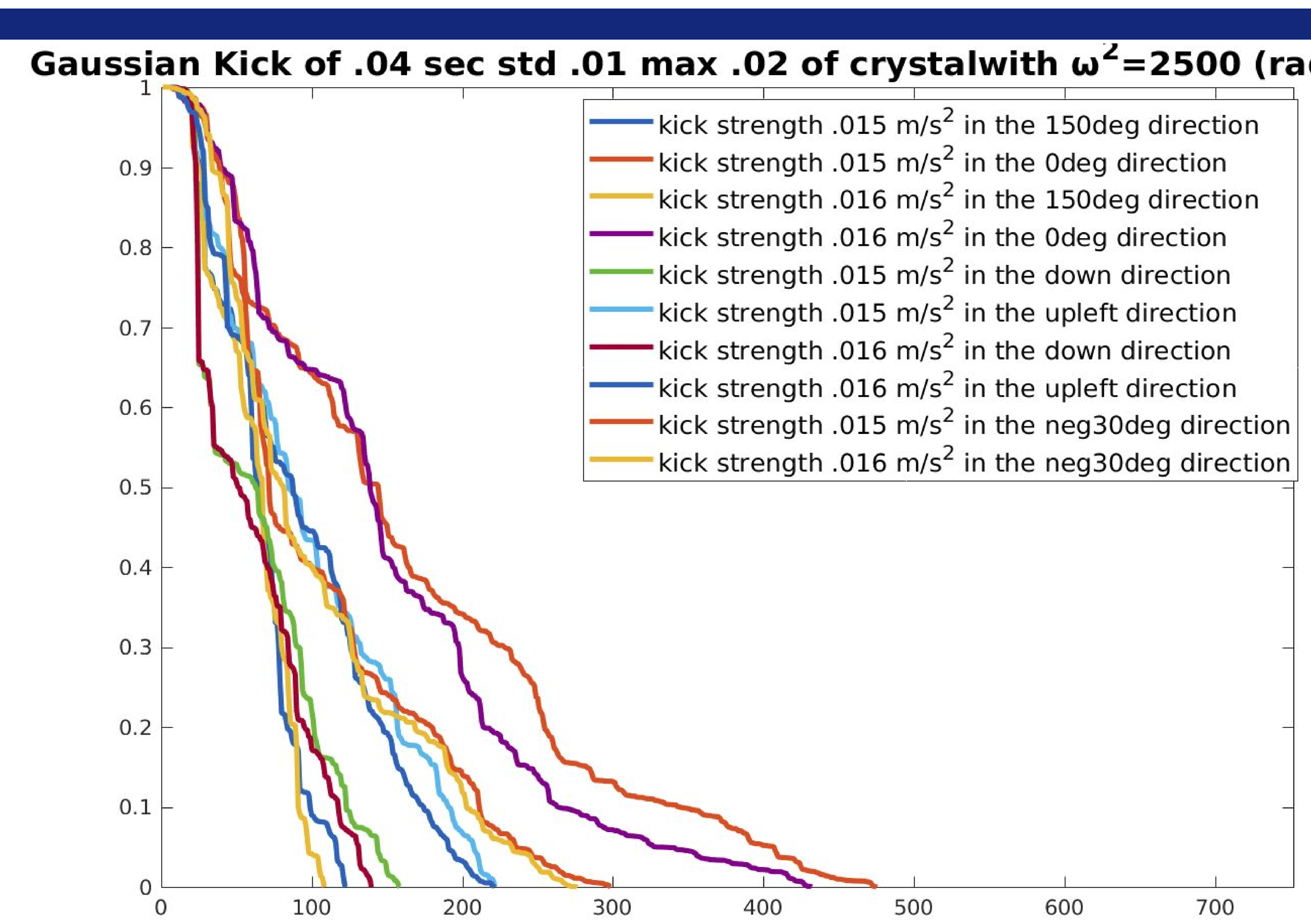


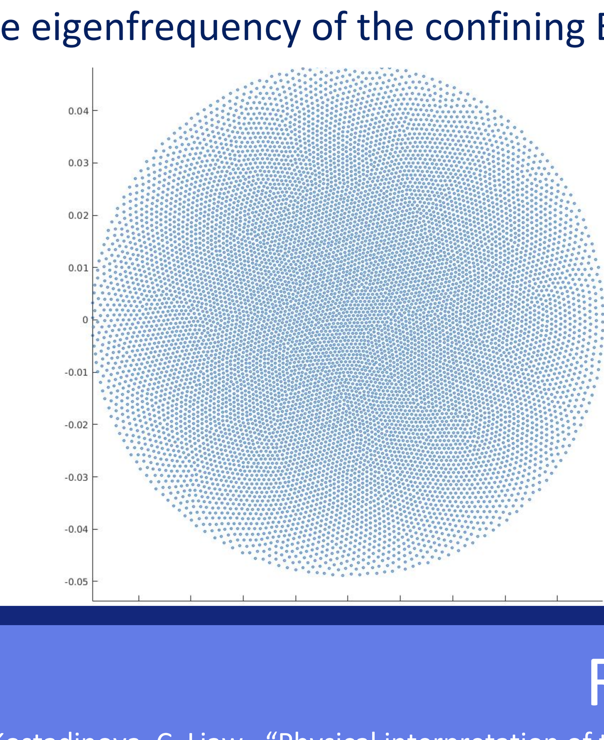

Illustration of a kick in progress

Different colors are different frames




Plot of distance to the cyclic subspace for localized state with crystalline disorder W= 0.0586 calculated in the radius of 0.04 m

ω is the eigenfrequency of the confining E-field, which exerts a linear restoring force. E=-kx



Plot of a plasma crystal

## References

[1] Eva Kostadinova, C. Liaw , "Physical interpretation of the spectral approach to delocalization in infinite disordered systems", Materials Research Express, Volume 3, Number 12, Dec 2016

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[3] Eva Kostadinova, K Busse, N Ellis, J Padgett, C D Liaw, L S Matthews, T W Hyde, "Delocalization in infinite disordered 2D lattices of different geometry", arXiv:1706.02800 [cond-mat.dis-nn], Jun 2017

[4] P. Hartmann, A. Douglass, J. C. Reyes, L. S. Matthews, T. W. Hyde, A. Kovacs, and Z. Donko, Crystallization Dynamics of a Single Layer Complex Plasma, Phys. Rev. Lett. 105, 115004 (2010).

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# Classifying States with the Spectral Approach to Anderson Localization

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# Outline

Anderson Localization
Spectral Theory
Cyclic Subspace-Time Evolution of states
Numerical Methods
2d materials and plasma crystals
Data
Conclusion

# Anderson Localization

Localization In tight binding model with random disorder

Extended states may become localized by self interference from phase changed parts of the wave reflected from crystal defects

Localization is defined as the wavefunction falling off in space faster than exponential decay

Delocalization is falling off nearly linearly-Ohm's law

for Classical waves MIT transition occurs when $l \sim \lambda$

For particles: purely quantum effect- classical analog is Einstein's brownian motion.

$$\vec{J} = -D\nabla\phi$$

# The Anderson Hamiltonian

varying onsite energies and hopping potential has the same effect as varying of one of these variables

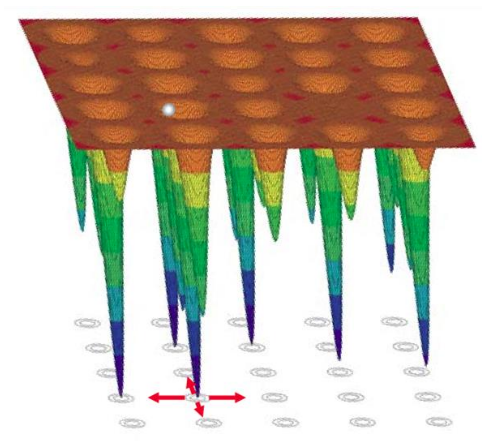
$$\hat{H}_\epsilon = -\Delta + \sum_{i\in\Gamma} \epsilon_i \delta_i |\delta_i\rangle$$

$$\hat{H} = \begin{pmatrix} \epsilon_1 & I & 0 & 0 \\ I & \epsilon_2 & I & 0 \\ 0 & I & \epsilon_3 & I \\ 0 & 0 & I & \epsilon_4 \end{pmatrix}$$

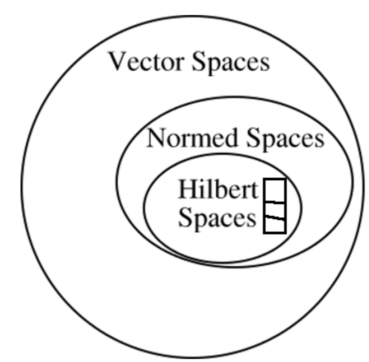
# Scaling theory

Edwards/ Thouless-2016 Nobel prize

- Scaling theory-application of periodic/antiperiodic boundary conditions. Peaks that consistently show up are localized. Model scaled up.



- Appears to fail in 2D
- Application of boundary conditions bounds the Hilbert space - may throw out some solutions



# Spectral Theory

Generalization of eigendecomposition to continuous basis
Can decompose operators into outer products of “cobasis vectors” (kets and bras)
Relies upon a change of measure in the completeness relation

$$\int f\,d\mu = \int \left[f\left[\delta(\xi-\lambda_1)+\delta(\xi-\lambda_2)+\cdots\right]+w(\xi)f(\xi)\right]d\xi$$

- Measure decomposed into 3 parts
- Obvious application to Schrodinger’s equation

# What part of the measure does an eigenvector correspond to?

It is shown generally that a vector corresponds to the continuous part if it is cyclic in the operator. In 2D,

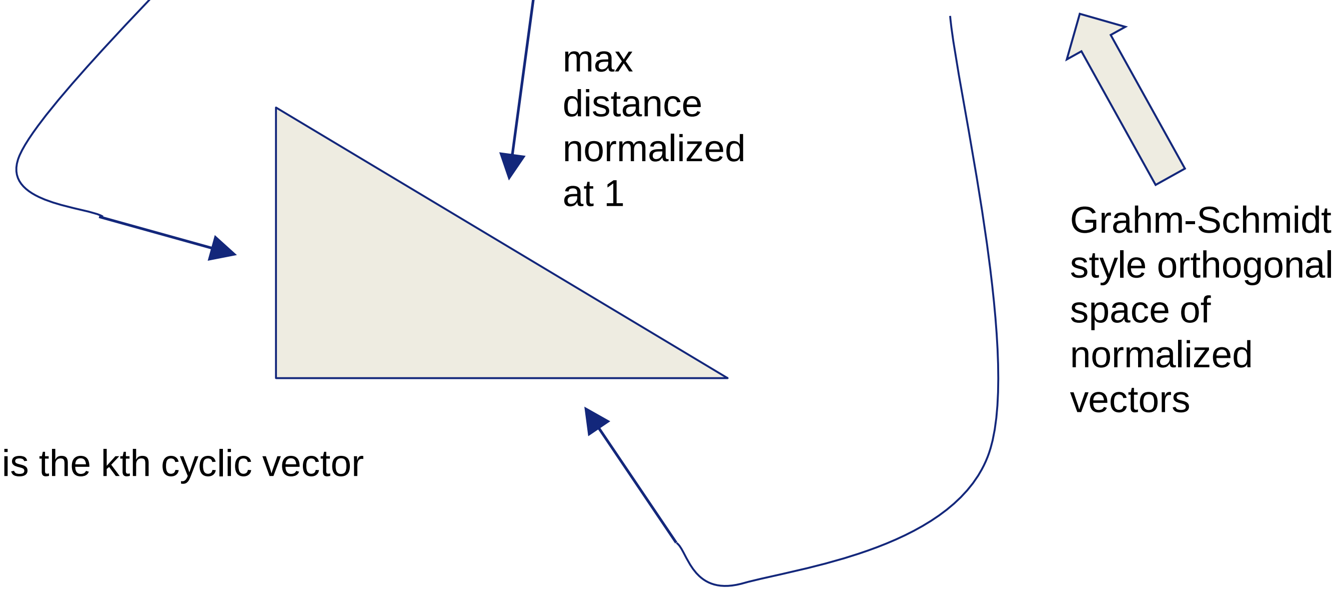
$$span\left\{|\psi\rangle, \hat{H}|\psi\rangle, \hat{H}^2|\psi\rangle, \cdots\right\} = \mathbb{R}^2$$

- makes sense - time translation

# Numerical Method

At each time step can calculate the distance to the cyclic subspace at each timestep and determine if it goes to zero in the end.

$$D^{n}_{\omega,c} = \sqrt{1 - \sum_{k=0}^{n} \frac{\langle m_k , v\rangle^2}{\|m_k\|_2^2}}$$




$m_k$ is the kth cyclic vector

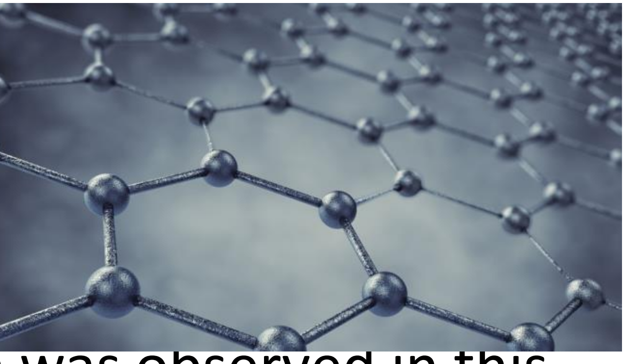
# 2D materials

In the mid 2000's we developed our first 2D materials(graphene). They have many applications. graphene is 2D graphite, it is

an incredibly good barrier

very strong

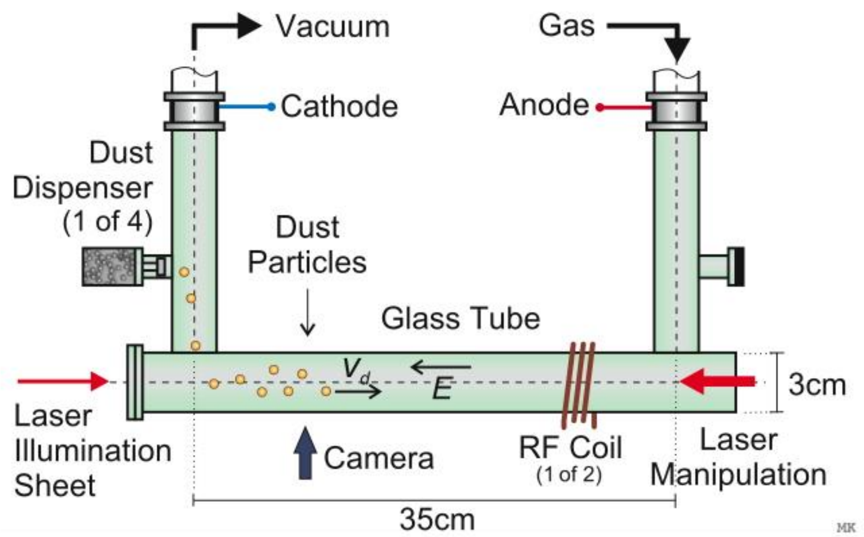
very flexible



In experimentation, delocalization was observed in this material although scaling theory always predicts localization for all disorders

# Plasma Crystals

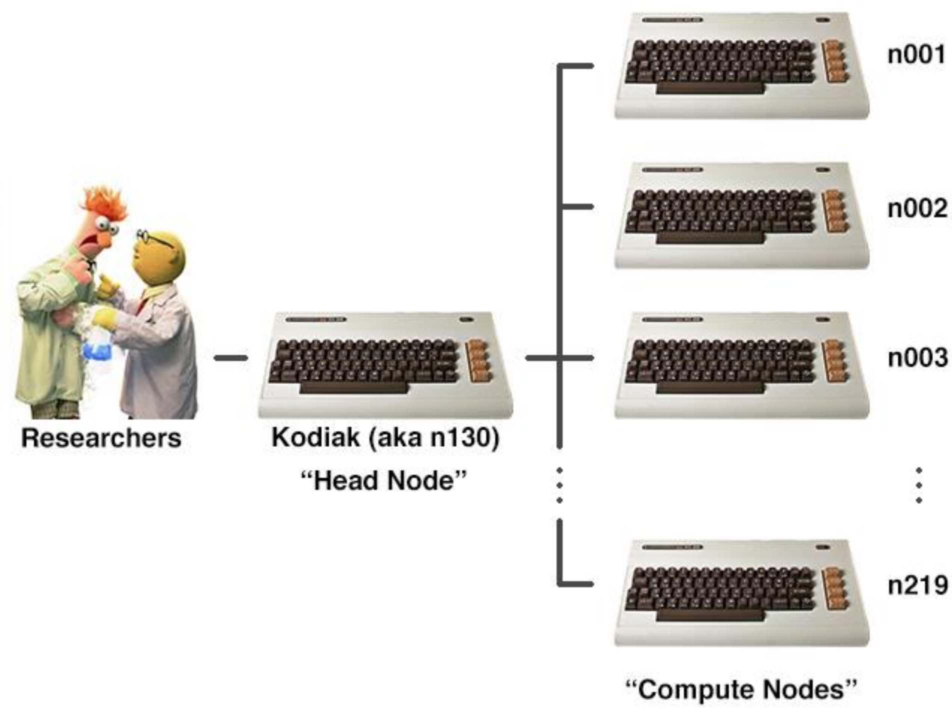
It is very difficult to manufacture graphene, but plasma crystals can be created in a PK4 mechanism with the same honeycomb structure by putting a small parabolic indentation held at a certain potential below the dust particles to act as radial confinement with a linear restoring force

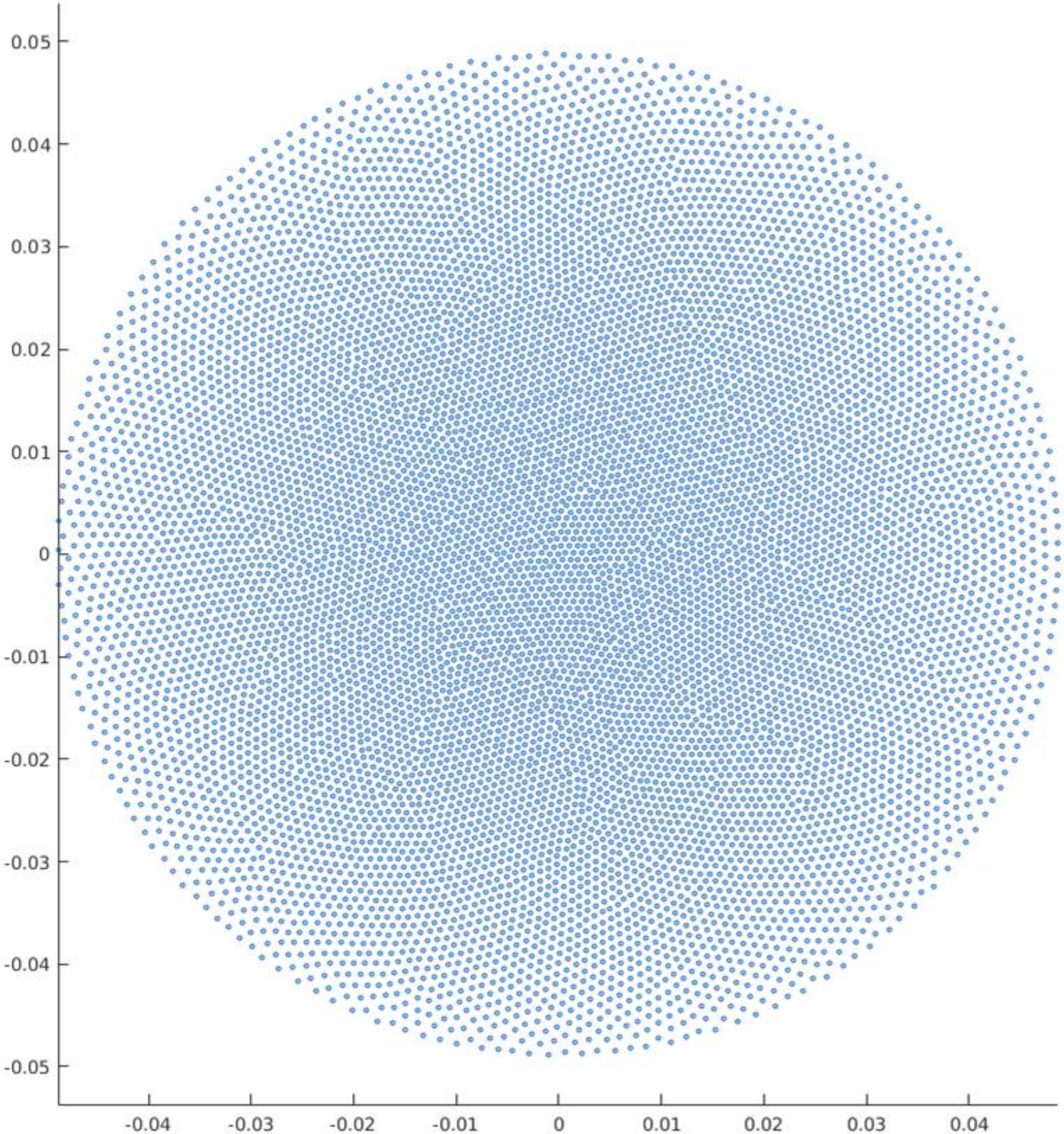


# Computer generated plasma crystals

A computer was used to generate plasma crystals at equilibrium with variation in confinements and particle size

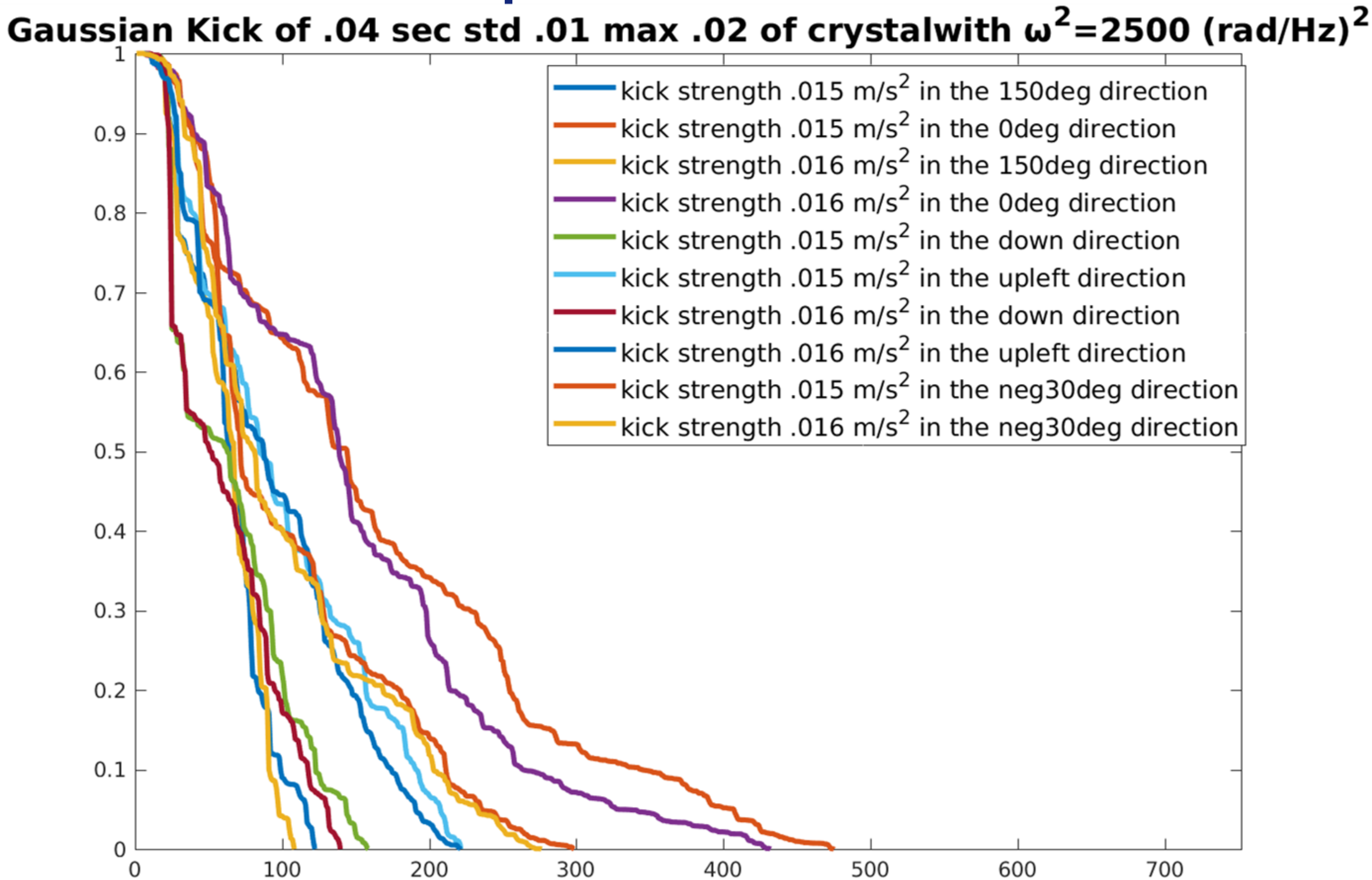
Then the particle nearest the center was kicked and the distance to the cyclic subspace at each timestep was calculated




0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04

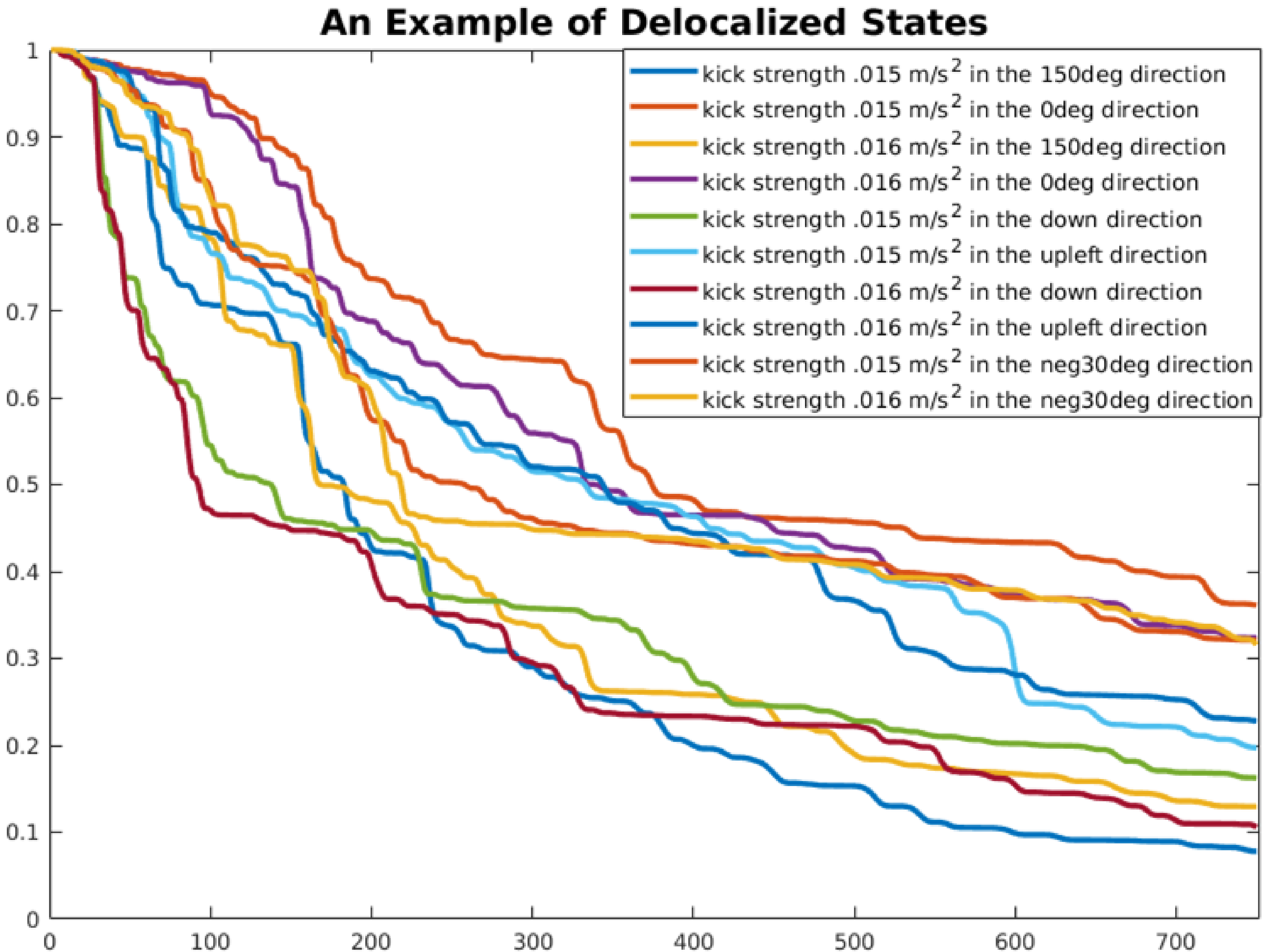
gaussian kick for .04 s at .016 $m/s^2$ directly up and to the left

# An example of localization




Plot of distance to the cyclic subspace for localized state with crystalline disorder W=0.0586 calculated in the radius of 0.04 m

w is the eigenfrequency of the confining E-field, which exerts a linear restoring force. E=-kx


An Example of Delocalized States
kick strength .015 m/s$^2$ in the 150deg direction
kick strength .015 m/s$^2$ in the 0deg direction
kick strength .016 m/s$^2$ in the 150deg direction
kick strength .016 m/s$^2$ in the 0deg direction
kick strength .015 m/s$^2$ in the down direction
kick strength .015 m/s$^2$ in the upleft direction
kick strength .016 m/s$^2$ in the down direction
kick strength .016 m/s$^2$ in the upleft direction
kick strength .015 m/s$^2$ in the neg30deg direction
kick strength .016 m/s$^2$ in the neg30deg direction
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
100
200
300
400
500
600
700

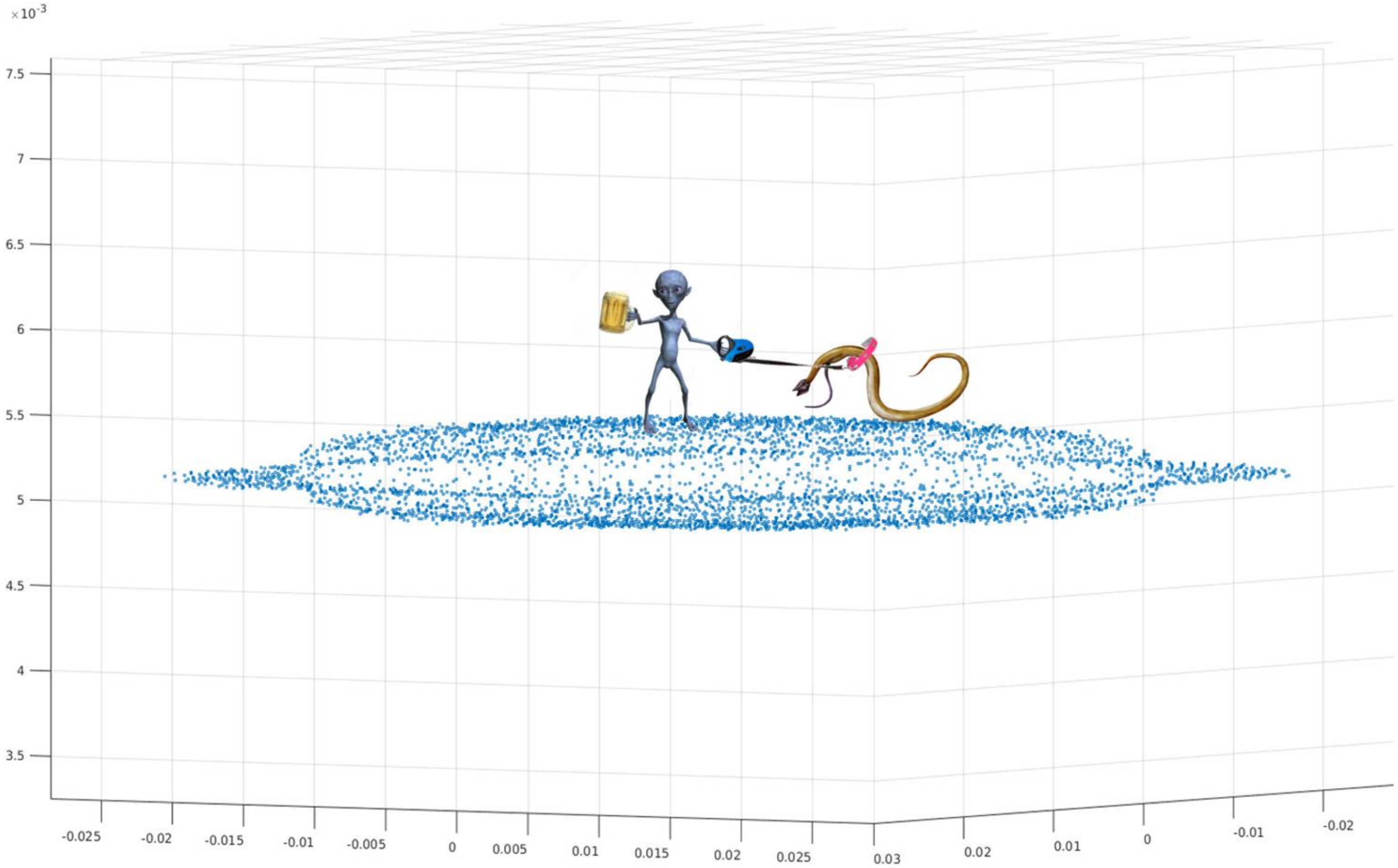
# Conclusion

The spectral method is valuable in the study of anderson localization in 2D by its capacity to find extended states. without throwing out solutions by bounding the space.

Plasma crystals are a useful 2D analogue of graphene.

Extended states do exist in 2D materials with nonzero W

# Thanks for your attention!



# Questions

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