

Dust Coagulation in the Solar Nebula using the Grand Tack Model

David Kelly, Augusto Carballido, Lorin Matthews, *Member, IEEE* and Truell W. Hyde, *Member, IEEE*

Abstract—To build a more accurate model of the formation of our solar system the Grand tack Model has been adopted as the strongest candidate to do so. To recreate the conditions present in the Solar Nebula we used a simulation to reproduce the affects of Magnetohydrodynamics to explain the driving force of the turbulence that drove the evolution of the protoplanetary disk and model the dust collisions that would have led to the formation of the terrestrial planets and explain the role Jupiter migration had in opening the gap between the gas giants and terrestrials and explaining how Jupiter migrated. Alpha values above .10 were observed and plots indicating the heavy influence of magnetic stresses contributed to this. One test for the dust coagulation code has not been passed yet.

I. INTRODUCTION

THE origins of how the planets in our solar system formed is surprisingly still a mystery contrary to what is generally known about planet formation. Most of these ideas are closes guesses at most and how our solar system truly began and the formation of the planets are elusive. The general understanding of the formation of the solar system is that the planets were formed in the accretion disk of a planetary nebula with the sun in the sun as the central mass, and over time the gas and dust accreted into larger objects as the system rotated forming larger and larger objects with the terrestrials closer to the sun and the gas giants toward the outside of the solar system. This model however only applies to our solar system from the fact that other systems have largely diverse configurations that this model doesn't explain well. As a result there have been different explanations trying to remedy this, one of them being Magnetohydrodynamics; an attempt to explain the driving force present that would accrete the gas and dust in a way to form the planets like they have.

Magnetohydrodynamics is the motion of objects in system which is best described as fluid-like motion driven by a changing magnetic field. The particles of the fluid are confined to the system and the magnetic field lines are attached somewhat to each particle so when the field changes it disrupts the flow of the particles and causes a magnetic turbulence similar to a

fluid; so in essence it is a study of magnetic fluids. In this case the source of the changing magnetic field would be the rotating sun and the field lines would extend out to the gas and dust in the protoplanetary disk. But as the disk rotates the field lines be come tangled, since they are fixed to the particles and the particles themselves are attracted to each other you have a tension like force between particles like a spring; but at the same time the particles closer to the solar mass are moving faster than those slightly far away so you now have a velocity gradient extending radially around the disk. As time goes on the particles exchange angular momentum with the rotating disk and fall in closer toward the center of the disk as consequence; this also pulls on the field lines bound to the particles and adds to the turbulence of the system which leads to an important point that will be discussed later in the introduction.

While the dust and gas are rotating around the disk the dust particles are colliding with each other, coagulating to form dust aggregates. The dust aggregates that formed will later on form larger more porous objects and then chondrules. The exact evolution of the dust depends on the collisions made i.e. dust with aggregates and aggregates with aggregates [1]. These particles are essentially the seeds from which all the solid structures in the solar system grow; these chondrules form asteroids which in turn from planetesimals and planets. Evidence of these chondurles are supported by their discovery in meteorite cores, and understanding them is key to understanding how planets are formed in protoplanetary disks.

Not all solar systems are like our own as previously stated, and leads to the question of why ours is the way it is. We have four terrestrial planets that orbit close to sun all within 1.5AU and farther out are the gas giants which extend beyond 25AU and a asteroid belt separating the rocky from the gaseous planets. One model shows promising validity of the likely formation and arrangements of our planets called The Grand-Tack Model. The old model of the solar system being formed from just accreting dust was not sufficient enough; simulations modeling this would get major errors the largest being the size of Mars, so there needed to be another explanation. It is well known that the gas giants were the first planets to form, removing most of the gas from the protopanetary disk; the proposition was that when Jupiter formed it migrated toward the sun; Saturn formed soon after and followed Jupiter in this migration where Jupiter pushed most of the material toward the sun to about 1.5AU. The two gas giants then migrated outward to their current locations as a property of hydrodynamics. This herding of solid material near the Sun is what is believed to have cause the formation of the terrestrials and Mars[2].

D. Kelly is an undergraduate studying Physics at University of West Florida, 11000 University Pkwy, Pensacola, FL, 32514 USA e-mail: djkl10@students.uwf.edu.

A. Carballido is a member of the Baylor CASPER research group and is with the Baylor University Department of Physics, One Bear Place, Waco, TX, 76798 USA e-mail: augusto_carballido@baylor.edu.

L. Matthews is an associate director of the Baylor CASPER research group and is with the Baylor University Department of Physics, One Bear Place, Waco, TX, 76798 USA e-mail: lorin_matthews@baylor.edu.

T. Hyde is an associate director of the Baylor CASPER research group and is with the Baylor University Department of Physics, One Bear Place, Waco, TX, 76798 USA e-mail: truell_hyde@baylor.edu.

All of these pieces of information are important to understanding the formation of our solar system and potentially other solar systems as well. Being able to understand how the chondrules were formed will substantiate the theory of protoplanetary disk formation and the evolution of the planets, and help lead to the understanding of more complicated mechanisms that were involved such as magnetohydrodynamics to build a complete model of solar system formation.

II. NUMERICAL METHODS

To answer the question of what happened in protoplanetary disk formation we have to build a simulation to recreate the conditions we believe to have caused the planets to form. We want to know specifically how and why Jupiter migrated and how it affected the coagulation of dust in the disk and how magnetohydrodynamics played a significant role in driving the turbulence in the disk. We will be simulating a protoplanetary disk with gas and dust under the influence of magnetohydrodynamic and other components of the dust particles motion. To do this we are using the Athena MHD code from Princeton University; it is an open source coding project many years in the making with a variety of parameters to simulate conditions in protoplanetary disks and interstellar medium. In conjunction we will also use a dust coagulation code to simulate how the dust particles collide and form larger aggregates during the disk evolution; these two codes will communicate and share data to construct an accurate model.

A. The Shearing Box

To simulate the evolution and rotation of the disk over time the protoplanetary disk is represented by a grid of boxes of a length x , a longer length y to represent the radial distances and a length z . To start a radial distance is chosen in the disk and the analysis is focused to a small patch in that distance[3]; the equations of motion, internal energy equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla (P + \frac{B^2}{8\pi}) + \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{4\pi \rho} - 2\boldsymbol{\Omega} \times \mathbf{v} + 2q\boldsymbol{\Omega}^2 x \hat{\mathbf{x}} - \boldsymbol{\Omega}^2 z \hat{\mathbf{z}}, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (3)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \quad (4)$$

These will be calculated in that box with periodic boundary conditions; so when events stretch outside of the box they will reappear on the other side. So the more boxes you have the better "resolution" your disk will have as it evolves over time. This shearing effect is an effective analog to represent the turbulent forces of the disk as seen in Figure 1. From this model we can calculate the relative velocities of the dust particles in the turbulent disk, this will be later used to do calculations in the coagulation code.

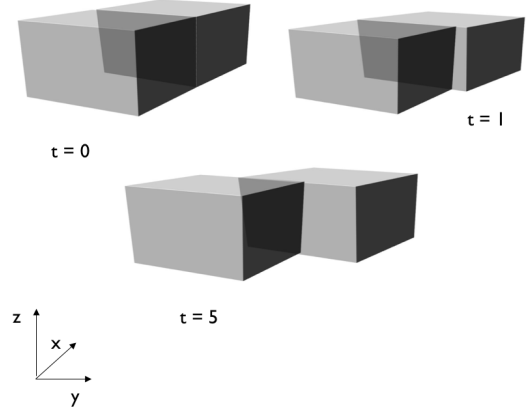


Fig. 1. A diagram representing the shearing box, in each picture representing the box at a time t to show the evolution of simulation over time

B. Planet Model

When running the ATHENA code at the beginning of the simulation a gravitational potential is placed in the center of the shearing box acting as Jupiter-like mass to model the behavior of the disk with Jupiter. During the introduction of this potential in the box it takes time for the gravitational interactions between the disk and the potential to reach an equilibrium and after that data from the simulation can be used.

C. Coagulation Method

To simulate the collision of the dust particles in the disk we are using the method from the paper via Ormel et al [4] the direct Monte-Carlo method. We have a box where we will have a certain number of particles; ideally the more the better so 20,000 would reproduce the results from that paper best. When two particles collide a new particle is formed and the system evolves like so over time. The method of the code is as follows. We have a number of particles in the system N distributed over a volume V , there are two different types of particles i and j , each with a different mass. The particles collide with a certain probability given by the calculated collision rate C_{ij} , which is determined by the equation:

$$C_{ij} = \frac{K_{ij}}{V} = \sigma_{ij}(a_i, a_j) \Delta v_{ij}(\tau_i, \tau_j) / \mathcal{V}(K) \quad (5)$$

K_{ij} being the collision kernel and v_{ij} being the gas velocities of the dust in the disk which are determined in the ATHENA code. A random number decides which particles collide making particle $i + j$ and after each collision the particle number is conserved by replacing the missing particle $N - 1$ with a random duplicate determined by a random number generator; this process is in turn repeated many times to produce a well defined distribution of masses over time. The collision are stored in a partial collision variable because each particle has its own collision rate which is summed, given by

this equation:

$$C_i \equiv \sum_{j=i+1}^N C_{ij}, i = 1, \dots, N-1 \quad (6)$$

and the total collision rates are given by this equation:

$$C_{tot} = \sum_i C_i \quad (7)$$

This process however can occupy a lot of processing power and memory because the matrix in which the collisions are performed contain all of the particle masses N and when you perform a collision all of the the change of the masses needs to be calculated for every particle, so the masses of N particles are recalculated N times creating an N^2 amount of information needing to be stored.

So to remedy this memory issue we have to use a different way to store and recalculate the particle masses, the best way is to only recalculate the masses and collision rates for the particles involved in a collision rather than for every particle present. To do this the collision kernel K_{ij} is calculated for every particle involved in the collision: particle i , j , the new particle $i+j$, and the random duplicate. Then we sum all of the pseudo K_{ij} 's to make a pseudo C_i , then store that calculation. We then use that pseudo C_i in the calculation of the C_i that will be used directly to determine the collision rates; this should only require the storage of N masses instead of N^2 .

1) *Tests:* For the coagulation code to be used it needs to pass two test, the constant and linear kernel test. The collision kernel from above have analytic solutions the constant and linear $K_{ij} = 1$ and $K_{ij} = \frac{1}{2}(m_i + m_j)$ respectively. Numerical solutions are generated by means of binning or "grouping" the masses from the collisions by representing each bin as if it were occupied by on particle to accurately represent the distribution of masses as time evolves. The test involves plotting the analytic solution represented by a solid curve against the numerical solutions represent by a dot; to pass the numerical solutions should almost overlap the analytic ones.

The coagulation method will work in conjunction with the Athena code using the calculated gas velocities which will be discussed in a later section.

D. Simulation

The simulation itself is run first with Athena used to calculate the gas velocities and then those values are used in the collision kernel of the coagulation code. The goal of the simulation is to recreate conditions in the protoplanetary disk as to observe the effect Jupiter migration had on the distribution of masses in the disk, shedding light on how that affected the dust coagulation that led to the development of the terrestrial planets.

1) *Grid Cells:* To simulate the solutions to the various equations of motion and forces acting in the shearing box they are divided into grid cells for simplicity of calculation. Rather than having one large box where all the calculations are simultaneously done the box is dived into cells in which the calculations are done in each one and they all work in

conjunction to give a complete picture of the disk, in the same way pixels work on a computer display to form an image or video. To simulate the turbulent shearing the grid cells are made with periodic boundary conditions represented by these equations:

$$\begin{aligned} f(x, y, z) &= f(x + L_x, y - q\Omega L_x t, z) && (x \text{ boundary}) \\ f(x, y, z) &= f(x, y + L_y, z) && (y \text{ boundary}) \\ f(x, y, z) &= f(x, y, z + L_z) && (z \text{ boundary}) \end{aligned} \quad (8)$$

These boundary conditions are so that a continuous shear flow is shown across all the boxes; so when a volume of fluid exits the box at one end it reappears with an adjust velocity and position on the other side of the box rather than directly across from the exit point.

2) *Run Time:* The run time for the simulation is long unfortunately, with the current job running for about 1500 hours and half way done. This code is large; so it is to be expected to take a long time and there isn't a way to speed up the process significantly enough. Though the dust coagulation code is being optimized so that it doesn't use as much memory and CPU time.

III. RESULTS

The ATHENA code is still running but enough time has passed to where the planet potential has reached an equilibrium with the disk and we can start plotting data, these plots are all from the 43rd orbit. In the plot from Figure 2 alpha has a rapid increase in the beginning of the simulation during the first orbit and starts to level off around the 10th orbit and at the 30th orbit reaches a more or less stable equilibrium. The Maxwell stresses follow the same pattern as alpha which will be discussed in the next section, and Reynolds stresses fluctuate the most out of the three throughout the 43 orbits but are more drastic during the first 10 orbits. This value of alpha is considered to be high, because it is over .10, alpha is a unit-less quantity represented by the equation:

$$\alpha = \frac{T_{MRI}}{\rho_0 c_s^2} = \alpha_{Re} + \alpha_{Max} \quad (9)$$

So as the turbulence within the disk increases the number alpha also increases, and an alpha of .20 is considered to be a large amount of turbulence.

The plot of the surface density of the disk in Figure 3 show that the planet potential in the center is surrounded by a gap-like region of low density with the regions of higher density being on both sides of the potential. There are long arms or streams of density extending from the potential and stretching across the gap to the regions of higher density, these arm-like structures and points of high density shall be addressed in the later section.

The results of the constant kernel test in Figure 4 were successful. The plots of the numerical solutions represented by dots closely follow the plotted curves of the analytical solutions represented by solid lines.

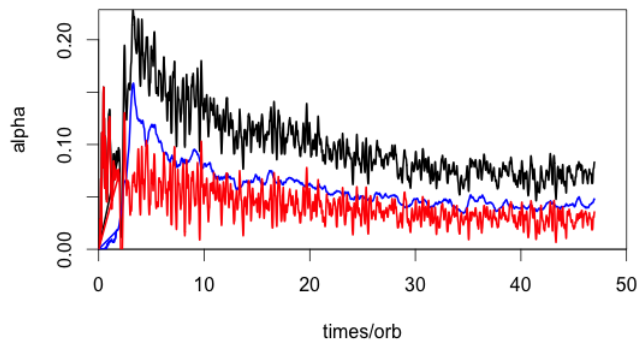


Fig. 2. A plot of alpha is shown in black: the ratio between the disk turbulence which is the sum of the Maxwell of Reynolds stresses, and the initial gas pressure[5], the normalized Maxwell stress or the magnetic stress is shown in blue, and the normalized Reynolds stress shown in red against the number of disk orbits.

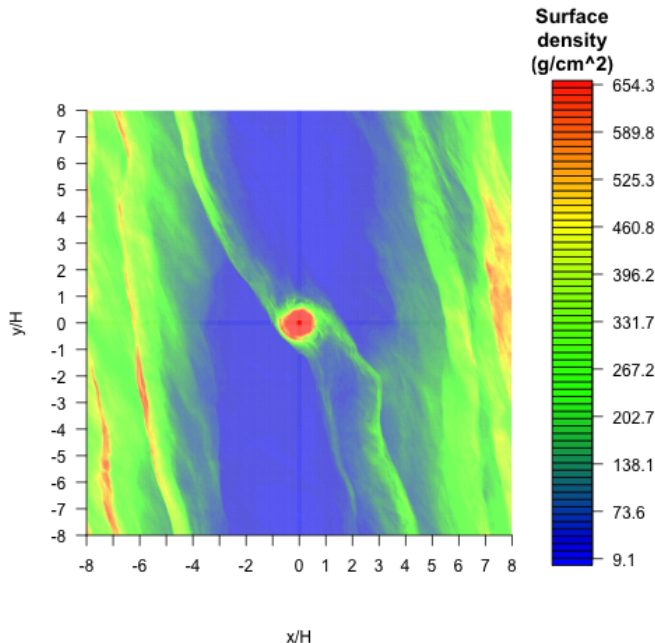


Fig. 3. A plot of the Surface Density in the shearing box during the 43rd orbit represented as a plane

IV. DISCUSSION

From the results shown in Figure 2 the plot of the Maxwell stress closely resemble the general shape of the plot of alpha, considering that alpha is the sum of the Maxwell and Reynolds stresses we can see that the Maxwell stress contributes considerably more to the value of alpha, which means that the magnetic stress contributes more to the turbulence in the disk than that of the Reynolds stress which is driven by hydrodynamics. The fluctuations through the plot of alpha can be attributed to the Reynolds stresses in the same way.

In Figure 3 the planet potential has these arm-like structures that extend to the edges of the gap which is typical for Jupiter

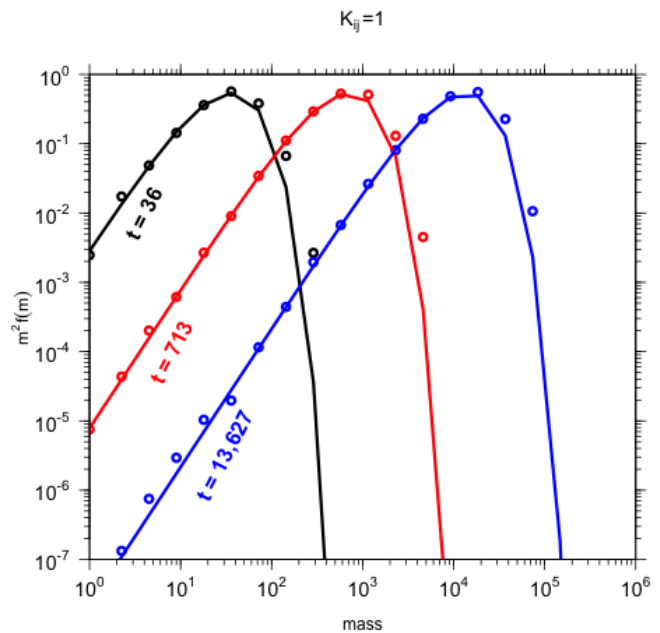


Fig. 4. The Constant Kernel test for the coagulation code, the collision kernel here is equal to 1. The analytical solutions are represented by the solid line curves and the numerical solutions are represented by dots. The different colored curves represent different time snapshots of the tests.

like masses in a protoplanetary disk. These structures are from the planet migrating in the rotating disk opening a gap creating these streams of higher density regions where angular momentum can be transferred between the gas and the planet. With the location of these streams in the midst of this region of low density it can have a "shock" effect on the gas from the rapid change in density possibly having an effect on the gas velocities, increasing them near the edges of the streams. The regions of high density are also likely places for the gas velocities to be higher in the disk.

V. CONCLUSION

Our simulation running the ATHENA code hasn't made 100 complete orbits like originally intended because of the long run time, but from the data of the 43rd orbit we can draw several conclusions. The value of alpha's evolution thought the simulation suggests that magnetic stresses are the primary cause of the turbulence of the disk rather than the hydrodynamic stresses. The size of alpha was large with a value of over .20 during the first 10 orbits, and mostly maintains a value of over .10 for the remainder of the orbits.

The surface density of the disk suggests regions of higher gas velocities near the edges of the gap of the planet potential and possibly in the streams extending from the potential because of the shock they experience from the difference in gas density from the gap.

The linear kernel still needs to be passed for the code to be used with ATHENA, but it is more a matter of "when" than "how". With that being said the future of this project is promising with only the issues of memory consumption, CPU time, and the linear kernel test to be resolved.

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project.

Lorin S. Matthews (M '09) was born in Paris, TX, in 1972. She received the B.S. and Ph.D. degrees in physics from Baylor University, Waco, TX, in 1994 and 1998, respectively.

She is currently an Assistant Professor with the Physics Department, Baylor University, where she is also with the Center for Astrophysics, Space Physics and Engineering Research. Previously, she was with Raytheon Aircraft Integration Systems, where she was the Lead Vibroacoustics Engineer on NASA's Stratospheric Observatory for Infrared Astronomy



David Kelly was born in Washington D.C in 1992. He is currently an undergraduate in physics at the University of West Florida, Pensacola, FL. He was a summer REU research student at the Center for Astrophysics Space Physics and Engineering Research at Baylor University, Waco, TX.

His current research interests include superconductor and protoplanetary disk modeling. Mr. Kelly is a member of the Society of Physics Students and the American Physical Society.



Truell W. Hyde (M '00) was born in Lubbock, TX, in 1956. He received the B.S. degree in physics and mathematics from Southern Nazarene University, Bethany, OK, in 1978 and the Ph.D. degree in theoretical physics from Baylor University, Waco, TX, in 1988.

He is currently with Baylor University, where he is the Director of the Center for Astrophysics, Space Physics and Engineering Research, a Professor of physics, and the Vice Provost for Research for the university. His research interests include space

physics, shock physics, and waves and nonlinear phenomena in complex (dusty) plasmas.



Augusto Carballido received a B.S. in Physics from the National Autonomous University of Mexico in 2002, and a PhD in astronomy from the University of Cambridge, U.K., in 2006.

His main interests are the formation of asteroids, cosmic dust, and the formation of the Moon. He is currently an Assistant Research Professor at Baylor University's Center for Astrophysics, Space Physics and Engineering Research.